Hadronic matrix elements for exclusive rare $B$-meson decays

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FCNC exclusive decays: $B \rightarrow K^{(*)} \ell^+\ell^-$, $B \rightarrow K^*\gamma$

- in Standard Model, $b \rightarrow s$ transitions, via $t$, $W$, $Z$ loops

\[
H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu),
\]

\[
O_{9(10)} = \frac{\alpha_{\text{em}}}{2\pi} [\bar{s}\gamma_\mu(1-\gamma_5)b] \ell\gamma^\mu(\gamma_5)\ell,
\]

$C_{9}(m_b) \simeq 4.4$, $C_{10}(m_b) \simeq -4.7$,

\[
O_7 = -\frac{m_b}{8\pi^2} [\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b] F^{\mu\nu},
\]

$C_7(m_b) \simeq -0.3$

$O_{1-6}$ - 4-quark, $O_8$ - quark-gluon also contribute!
B → K(∗) ℓ+ ℓ−, B → K*γ, the observables

decay amplitudes:
\[ A(B \to K(∗)\ell^+\ell^-) = \langle K(∗)\ell^+\ell^- \mid H_{\text{eff}} \mid B \rangle \]

all decay distributions, BR’s, asymmetries, etc.
determined by
\[ |A(B \to K(∗)\ell^+\ell^-)|^2 \times \{ \text{phase space} \} \]

\[ A(B \to K^*γ) = \langle K(∗)γ \mid H'_{\text{eff}} \mid B \rangle \]

important contributions are the same as in
\[ A(B \to K(∗)\ell^+\ell^-) \text{ at } q^2 = 0, \]

\[ B \to K(∗)\ell^+\ell^- \text{ contributes to inclusive } B \to X_s\ell^+\ell^-, \text{ but !} \]
in the theory, the inclusive width is defined and treated differently.

For exclusive decays we need hadronic matrix elements
**B → K, K* form factors**

- contributions of $O_{9,10}$ and $O_7$ factorize, e.g.,

$$\langle K^*(p)|C_9 O_9|B(p+q)\rangle$$

$$= \frac{\alpha_{em}}{2\pi} C_9 \langle K^*(p)|\bar{s}\gamma_\mu(1 - \gamma_5)b|B(p+q)\rangle(\bar{\ell}\gamma_\mu\ell)$$

\[\Downarrow\]

**B → K and B → K* form factors**

- form factors depend on $q^2 = (p_\ell^+ + p_\ell^-)^2$,

$$0 < q^2 < (m_B - m_{K^(*)})^2 \text{ in } B \to K^(*) \ell^+\ell^-, \ q^2 = 0 \text{ in } B \to K^*\gamma$$

- dominated by nonperturbative quark-gluon interactions,
Calculating the form factors in QCD

- Lattice QCD with growing accuracy:
  currently, \( B \rightarrow \pi, K \) at large \( q^2 \), \( n_f = 3 \),
  \( B \rightarrow \rho, K^* \) only quenched

- non-lattice method: QCD Light-cone sum rules (LCSR)
  - outline:
    correlation function in QCD via light-cone OPE
    \( \Rightarrow \) hadronic dispersion relation
    \( \Rightarrow \) ground state contribution
    \( \Rightarrow \) form factor
  - main input:
    quark masses, \( \alpha_s \),
    light-cone distribution amplitudes (DA’s) of \( \pi, K, B, \ldots \)
Status and accuracy of LCSR calculations

- the region $q^2 \leq 12 - 15 \text{ GeV}^2$ accessible, complementing the lattice FF’s
- $B \rightarrow \pi, K$ form factors
  

- estimated uncertainties for $B \rightarrow \pi, K \pm (12 - 15)\%$

- $B_{(s)} \rightarrow \rho, \omega, K^*, \phi$ form factors, DA’s of $K^*$ and $\rho$
  [P. Ball, R. Zwicky (2005)]

- $B \rightarrow \pi, K, \rho, K^*, ...$

  LCSR with B-meson distribution amplitudes

  [A. K., Th. Mannel, N. Offen (2005)]

  LCSR in SCET [F. De Fazio, Th. Feldmann and T. Hurth (2006)]
\[ B \rightarrow K, K^{(*)} \] form factors from LCSR


<table>
<thead>
<tr>
<th>form factor</th>
<th>( F^i_{BK(*)}(0) )</th>
<th>( b^i_1 )</th>
<th>( B_s(J^P) )</th>
<th>input at ( q^2 &lt; 12 \text{ GeV}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f^+_{BK} )</td>
<td>( 0.34^{+0.05}_{-0.02} )</td>
<td>(-2.1^{+0.9}_{-1.6} )</td>
<td>( B_s^*(1^-) )</td>
<td>no pole</td>
</tr>
<tr>
<td>( f^0_{BK} )</td>
<td>( 0.34^{+0.05}_{-0.02} )</td>
<td>(-4.3^{+0.8}_{-0.9} )</td>
<td></td>
<td>( B_s^*(1^-) )</td>
</tr>
<tr>
<td>( f^T_{BK} )</td>
<td>( 0.39^{+0.05}_{-0.03} )</td>
<td>(-2.2^{+1.0}_{-2.00} )</td>
<td></td>
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</tr>
<tr>
<td>( V^{BK*} )</td>
<td>( 0.36^{+0.23}_{-0.12} )</td>
<td>(-4.8^{+0.8}_{-0.4} )</td>
<td>( B_s^*(1^-) )</td>
<td></td>
</tr>
<tr>
<td>( A^{BK*}_1 )</td>
<td>( 0.25^{+0.16}_{-0.10} )</td>
<td>( 0.34^{+0.86}_{-0.80} )</td>
<td>( B_s(1^+) )</td>
<td></td>
</tr>
<tr>
<td>( A^{BK*}_2 )</td>
<td>( 0.23^{+0.19}_{-0.10} )</td>
<td>(-0.85^{+2.88}_{-1.35} )</td>
<td>( B_s(1^+) )</td>
<td>LCSR with B DA's</td>
</tr>
<tr>
<td>( A^{BK*}_0 )</td>
<td>( 0.29^{+0.10}_{-0.07} )</td>
<td>(-18.2^{+1.3}_{-3.0} )</td>
<td>( B_s(0^-) )</td>
<td></td>
</tr>
<tr>
<td>( T^{BK*}_1 )</td>
<td>( 0.31^{+0.18}_{-0.10} )</td>
<td>(-4.6^{+0.81}_{-0.41} )</td>
<td>( B_s^*(1^-) )</td>
<td></td>
</tr>
<tr>
<td>( T^{BK*}_2 )</td>
<td>( 0.31^{+0.18}_{-0.10} )</td>
<td>(-3.2^{+2.1}_{-2.2} )</td>
<td>( B_s(1^+) )</td>
<td></td>
</tr>
<tr>
<td>( T^{BK*}_3 )</td>
<td>( 0.22^{+0.17}_{-0.10} )</td>
<td>(-10.3^{+2.5}_{-3.1} )</td>
<td>( B_s(1^+) )</td>
<td></td>
</tr>
</tbody>
</table>

using BCL [Bourrely, Caprini, Lellouch(2008)] parameterization, \( b_1 \)-slope parameter.
Charm-loops and other complications

- a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\ell\ell)$ “mimicking FCNC”
- Charm-loop effect:

![diagram]

- similar $u, d, s, c, b$-quark loops ($O_{3-6}$ operators),
- $u$-loops from $O_{1,2}^u$ (CKM suppressed in $b \rightarrow s$),
- "weak annihilation" $\oplus$ photon emission
- factorization is lost!
- $A(B \rightarrow K^{(*)}\ell^+\ell^-)$ include additional hadronic matrix elements, not simply form factors
Charm loop turns charmonium

- at $q^2 \rightarrow m_{J/\psi}$, ... an on-shell hadronic state:
  $B \rightarrow J/\psi K \otimes J/\psi \rightarrow \ell^+\ell^-$

- other $\psi$-levels (charmonia with $J^P = 1^-$),
  open-charm states $B \rightarrow \bar{D}DK \rightarrow K\ell^+\ell^-$,
  ($\bar{c}c$ states with the masses up to $m_B - m_{K^*}$)

- $J/\psi$ and $\psi(2S)$ bins are subtracted from the $q^2$-distribution data in $B \rightarrow K^{(*)}\ell^+\ell^-$

- the effect of intermediate virtual $\bar{c}c$ states remains at
  $q^2 \ll m_{J/\psi}^2$ (nonperturbative at $q^2 \sim 4m_c^2$)
Charm-loop in $B \rightarrow K^{(*)} \ell^+ \ell^-$

- factorizable c-quark loop
  \[ C_9 \rightarrow C_9 + (C_1 + 3C_2)g(m_c^2, q^2) \]

- perturbative gluons \rightarrow (nonfactorizable) corrections
  being factorized in $O(\alpha_s)$
  and added to $C_9$


- taking into account the soft gluons (low-virtuality, nonvanishing momenta) emitted from the c-quark loop

- light-cone expansion \Rightarrow nonlocal effective $\bar{s}Gb$ operator
  \[ \sim 1/(4m_c^2 - q^2) \]-suppression

- the $B \rightarrow K^{(*)}$ hadronic matrix elements of this operator calculated
  using the same LCSR method as for $B \rightarrow K^{(*)}$ form factors
Charm-loop effect in $B \rightarrow K \ell^+ \ell^-$ in terms of $\Delta C_9$

- the effective coefficient $C_9(\mu = m_b) \simeq 4.4$
- a process-dependent correction to be added:

$$\Delta C_9^{(\bar{c}c, B \rightarrow K)}(q^2) = (C_1 + 3C_2) g(m_c^2, q^2) + 2C_1 \frac{32\pi^2}{3} \frac{\tilde{A}(q^2)}{f_{BK}(q^2)}$$

$$\Delta C_9(0) = 0.17^{+0.09}_{-0.18}, \quad (\mu = m_b = 4.2\text{GeV})$$
Charm-loop effect for $B \rightarrow K^* \ell^+ \ell^-$

- Factorizable part determined by the three $B \rightarrow K^*$ form factors $V^{BK^*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- Three kinematical structures for the nonfactorizable part:

$$\Delta C_9^{(\bar{c}c,B\rightarrow K^*,V)}(q^2) = (C_1 + 3C_2) g(m_c^2, q^2)$$

$$- 2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*}) \tilde{A}_V(q^2)}{q^2 V^{BK^*}(q^2)}$$

- Nonfactorizable part enhances the effect, $1/q^2$ factor

$$\Delta C_9^{(\bar{c}c,B\rightarrow K^*,V)}(1.0\text{GeV}^2) = 0.7^{+0.6}_{-0.4}$$
$$\Delta C_9^{(\bar{c}c,B\rightarrow K^*,A_1)}(1.0\text{GeV}^2) = 0.8^{+0.6}_{-0.4}$$
$$\Delta C_9^{(\bar{c}c,B\rightarrow K^*,A_2)}(1.0\text{GeV}^2) = 1.1^{+1.1}_{-0.7}$$
Charm-loop effect in $B \rightarrow K^*\gamma$

- By-product of our calculation for $B \rightarrow K^*\ell^+\ell^-$ at $q^2 = 0$
- Factorizable part vanishes, nonfactorizable part yields a correction to $C_{1,2}^{\text{eff}}(m_b) \approx -0.3$ in the two inv. amplitudes:

$$C_{1,2}^{\text{eff}} \rightarrow C_{1,2}^{\text{eff}} + \left[ \Delta C_{7}^{(\bar{c}c, B \rightarrow K^*\gamma)} \right]_{1,2},$$

$$\left[ \Delta C_{7}^{(\bar{c}c, B \rightarrow K^*\gamma)} \right]_{1} \approx \left[ \Delta C_{7}^{(\bar{c}c, B \rightarrow K^*\gamma)} \right]_{2} = (-1.2^{+0.9}_{-1.6}) \times 10^{-2},$$

- The previous results in the local OPE limit, LCSR with $K^*$ DA:

$$\left[ \Delta C_{7}^{(\bar{c}c, B \rightarrow K^*\gamma)} \right]_{BZ}^{(\bar{c}c, B \rightarrow K^*\gamma)} = (-0.39 \pm 0.3) \times 10^{-2},$$

$$\left[ \Delta C_{7}^{(\bar{c}c, B \rightarrow K^*\gamma)} \right]_{BZ}^{(\bar{c}c, B \rightarrow K^*\gamma)} = (-0.65 \pm 0.57) \times 10^{-2}. \quad (1)$$

[P. Ball, G. W. Jones and R. Zwicky (2007)]

- Our result in the local limit is closer to 3-point sum rule estimate: [A.K., G. Stoll, R. Rueckl, D. Wyler (1997)]
Accessing large $q^2$ with dispersion relation

- analyticity of the hadronic matrix element in $q^2$, dispersion relation:

$$\mathcal{H}^{(B \to K)}(q^2) = \mathcal{H}^{(B \to K)}(0) + q^2 \left[ \sum_{\psi=J/\psi,\psi(2S)} f_{\psi} A_{B\psi K} \frac{m_{\psi}^2 (m_{\psi}^2 - q^2 - i m_{\psi} \Gamma_{\psi}^{tot})}{m_{\psi}^2 (m_{\psi}^2 - q^2 - i m_{\psi} \Gamma_{\psi}^{tot})} \right] + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)}$$

- absolute values of the residues $|f_{\psi} A_{B\psi K}|$ from exp. data
- the integral over $\rho(s)$ fitted as an effective pole
- using LCSR result at $q^2 \ll 4m_c^2$ as an input
- the whole $q^2 \leq m_{\psi(2S)}^2$ region accessed
- need more data on $B \to \psi K(\ast), B \to \bar{D}DK(\ast)$
Influence on the observables for $B \rightarrow K \ell^+ \ell^-$

- adding the calculated $\Delta C_9(q^2)$ to the $(C_9)_{FCNC}$ in the decay amplitude
- differential distribution in $q^2$ with (solid) and without (dashed) charm-loop effect

![Graph showing the differential distribution in $q^2$ with (solid) and without (dashed) charm-loop effect.]

- green shaded - estimated uncertainty
- (of the LCSR result for soft-gluon effect)
Observables for $B \rightarrow K^* \ell^+ \ell^-$

- predicted shape in $q^2$ with (solid) and without (dashed) charm-loop effect

\[ N(q^2) = \frac{\overline{B}_0 \rightarrow \overline{K}^0 \ell^+ \ell^-}{B_0 \rightarrow K^0 \ell^+ \ell^-} \]

decay width $q^2$=distribution normalized at $q^2 = 1.0 \text{ GeV}^2$

no uncertainties of form factors shown!
Outlook and preliminary results

- work in progress: all soft-gluon effects (penguin operators, WA), improving $B \to V$ form factors, adding the hard-gluon nonfactorizable contributions from QCDF

- FB asymmetry mainly shifted by the hard-gluon effects
- towards SM prediction for the observables in $B_s \to P(V)\ell^+\ell^-$ at $q^2 \leq m_{\psi}^2$ including all hadronic effects