$B \rightarrow D^{(*)}$ form factors from QCD sum rules

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Outline

1. Theory of $B \rightarrow D^{(*)}$ decays
   - Introduction: Determination of $|V_{cb}|$
   - Use of Heavy Quark Symmetry

2. QCD Sum rules
   - Basic idea of QCD sum rules
   - Light cone sum rules
   - Distribution amplitudes

3. Results and Discussion
   - Numerical results
   - Summary and Outlook
1 Theory of $B \to D^{(*)}$ decays
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Determination of $|V_{cb}|$

Current status of the determination of $|V_{cb}|$ [Kowalewski, Mannel 2008]

Independent considerations of exclusive and inclusive decays
(important cross-checking):

- Exclusive decays:
  $$ |V_{cb}|_{excl} = (38.6 \pm 1.3) \cdot 10^{-3} $$

- Inclusive decays:
  $$ |V_{cb}|_{incl} = (41.6 \pm 0.6) \cdot 10^{-3} $$

- Average:
  $$ |V_{cb}| = (41.2 \pm 1.1) \cdot 10^{-3} $$
Determination from exclusive decays

- Relative precision at about 3%.
- Consideration of the semileptonic decays $B \rightarrow D$ and $B \rightarrow D^*$. 
- $B \rightarrow D$ more difficult to measure than $B \rightarrow D^*$ in that edge of phase space (small rate, high background).

→ Subject of this talk.
→ See also talk by N. Uraltsev.
Determination from inclusive decays

- Relative precision at below 2%.
- Most precise determination at the moment.
- Theoretical tool used here: Heavy Quark Expansion
- Uncertainty mainly comes from theoretical ignorance of higher order and nonperturbative corrections.
Determination of $|V_{cb}|$ from inclusive decays

Determination from inclusive decays

- Relative precision at below 2%.
- Most precise determination at the moment.
- Theoretical tool used here: Heavy Quark Expansion
- Uncertainty mainly comes from theoretical ignorance of higher order and nonperturbative corrections.
- Work in our group by B. Dassinger, R. Feger, Th. Mannel, S. Turczyk, N. Uraltsev, ...

→ See talk by S. Turczyk for more details.
Exclusive decay $B \rightarrow D^{(*)}$

Here the important dynamics comes from the transition matrix elements beside $V_{cb}$ and kinematical factors.
Exclusive decay $B \rightarrow D^{(*)}$

→ Determination of $V_{cb}$ from the experimental measured spectrum:

$$\frac{d\Gamma_{(B\rightarrow D\ell\nu_\ell)}(q^2)}{dq^2} \sim |V_{cb}|^2 \cdot |\langle D|\bar{c}\gamma^\mu(1-\gamma_5)b|B\rangle|^2$$

→ Here the important dynamics comes from the transition matrix elements beside $V_{cb}$ and kinematical factors.
Form factors

These are parametrized by form factors:

\[
\langle D(p')|\bar{c} \gamma^\mu b|B(p)\rangle = (p^\mu + p'^\mu) \cdot f_+(q^2) + (p^\mu - p'^\mu) \cdot f_-(q^2)
\]

\[
\langle D^*(p',\epsilon)|\bar{c} \gamma^\mu b|B(p)\rangle = \frac{2V(q^2)}{m_B + m_{D^*}} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu p^\alpha p'^\beta (q^\mu = p^\mu - p'^\mu)
\]

\[
\langle D^*(p',\epsilon)|\bar{c} \gamma^\mu \gamma_5 b|B(p)\rangle = i\epsilon^{\mu} (m_B + m_{D^*}) \cdot A_1(q^2)
\]

\[
- i(\epsilon^\ast \cdot q) (p^\mu + p'^\mu) \cdot \frac{A_2(q^2)}{m_B + m_{D^*}}
\]

\[
- i(\epsilon^\ast \cdot q) (p^\mu - p'^\mu) \cdot \frac{2m_{D^*}}{q^2} (A_3(q^2) - A_0(q^2))
\]

→ The complete expression for the decay rate \((m_\ell = 0)\):

\[
\frac{d\Gamma_{(B \rightarrow D \ell \nu_\ell)}(q^2)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \cdot |f_+(q^2)|^2 \cdot \left((q^2 - m_B^2 - m_{D^*}^2)^2 - 4m_B^2 m_{D^*}^2\right)^{3/2}
\]
Form factors and nonperturbative methods

\[
\frac{d\Gamma_{(B \to D\ell\nu_\ell)}(q^2)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \cdot |f_+(q^2)|^2 \cdot \left( (q^2 - m_B^2 - m_D^2)^2 - 4m_B^2 m_D^2 \right)^{3/2}
\]

\[V_{cb} \text{ and form factors always appear together.}\]

\[\to \text{A theoretical determination of the form factors is necessary.}\]
Form factors and nonperturbative methods

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\[B,D\text{-Mesons are bound states of heavy } b,c\text{-quarks and light antiquarks}\]

\[\text{(strong quark-gluon-interactions, confinement):}\]

\[\text{Matrix elements are not perturbatively calculable!}\]
Form factors and nonperturbative methods

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\[B, D\text{-Mesons are bound states of heavy } b, c\text{-quarks and light antiquarks}\]

\text{(strong quark-gluon-interactions, confinement):}

\[\text{Matrix elements are not perturbatively calculable!}\]

\[\rightarrow \text{Use of nonperturbative methods:}\]

\textit{QCD-Sum rules, Lattice-QCD}\n
\[\text{and}\]

\[\rightarrow \text{Use of symmetries:}\]

\textit{Heavy Quark Symmetry (HQS)
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Heavy Quark Effective Theory (HQET)

Important tool for handling heavy quarks: Heavy Quark Effective Theory (HQET)

- Formulation of QCD as an effective theory for heavy quarks: \( m_Q \gg \Lambda_{QCD} \)
- Definition: Meson has four-velocity \( v^\mu = \frac{p^\mu_M}{m_Q} \)

  \[ \rightarrow \text{Heavy Quark carries momentum} \quad p_Q^\mu = m_Q v^\mu + \mathcal{O}(\Lambda_{QCD}) \]

- Parametrization and integrating out of the heavy degrees of freedom \( \sim m_Q \) of the meson

\[ \rightarrow \text{Expansion of the QCD-Lagrangian and the meson states as series in } \frac{1}{m_Q} : \]

  - Operators: \( Q(x) = e^{-im_Q v \cdot x} h_v(x) + \mathcal{O} \left( \frac{1}{m_Q} \right) \)
  - Meson-states: \( |M\rangle = |H_v\rangle + \mathcal{O} \left( \frac{1}{m_Q} \right) \)
Spin-flavour-symmetry for heavy quarks

The light degrees of freedom in the meson interact with the heavy quark only at momentum scales $\sim \Lambda_{QCD} \ll m_Q$.

*Beside of corrections of order $\sim \mathcal{O}(\frac{1}{m_Q})$ the spin-flavour-symmetry holds:*

The light degrees of freedom in the meson neither notice the flavour nor the spin of the heavy quark.
Spin-flavour-symmetry for heavy quarks

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Beside of corrections of order $\sim \mathcal{O}\left(\frac{1}{m_Q}\right)$ the spin-flavour-symmetry holds:

The light degrees of freedom in the meson neither notice the flavour nor the spin of the heavy quark.

$\Rightarrow$ In their dynamics the mesons $B, B^*, D, D^*$ are not different.

$\Rightarrow$ All form factors can be expressed by a single function, the Isgur-Wise-function $\xi(w)$

$$w = \mathbf{v} \cdot \mathbf{v'} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

Normalization: $\xi(1) = 1$
Spin-flavour-symmetry for heavy quarks

- Alternative definition of the form factors more useful in HQET:

\[
\frac{\langle D(v') | V^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_D}} = h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu
\]

\[
\frac{\langle D^*(v', \epsilon) | V^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_D^*}} = h_V(w)\epsilon^\mu\nu\alpha\beta \epsilon^{*\nu}_{\nu}\epsilon_{\alpha\beta}
\]

\[
\frac{\langle D^*(v', \epsilon) | A^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_D^*}} = -i\epsilon^{*\mu}(w + 1)h_{A_1}(w) + i(\epsilon^{*\cdot q})\nu^\mu h_{A_2}(w) + i(\epsilon^{*\cdot q})\nu'\nu h_{A_3}(w)
\]
Spin-flavour-symmetry for heavy quarks

- Alternative definition of the form factors more useful in HQET:

\[
\frac{\langle D(v') | V^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_D}} = h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu \quad \frac{\langle D^*(v', \epsilon) | V^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_D^*}} = h_{V}(w)\epsilon^{\mu \nu \alpha \beta} \epsilon^*_\nu v^\alpha v^\beta
\]

\[
\frac{\langle D^*(v', \epsilon) | A^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_D^*}} = -i \epsilon^* \mu (w + 1) h_{A_1}(w) + i (\epsilon^* \cdot q) v^\mu h_{A_2}(w) + i (\epsilon^* \cdot q) v'^\mu h_{A_3}(w)
\]

- The heavy-quark-symmetry now predicts at \(w = 1\):

\[
h_+(1) = h_V(1) = h_{A_1}(1) = h_{A_3}(1) = 1 + \mathcal{O}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2 \quad \rightarrow \text{„Luke's theorem“}
\]

\[
h_-(1) = h_{A_2}(1) = 0 + \mathcal{O}\left(\frac{1}{m_c}, \frac{1}{m_b}\right)
\]

\(\rightarrow\) High theoretical precision for determination of \(|V_{cb}|\) at \(w = 1\).
But: phase space vanishes at \( w = 1 \)

→ more difficult to compare to experiment.
→ extrapolation of experimental data to \( w = 1 \) is done.

It is important to know the shape of the form factor.

\[
\eta_{A}, \eta_{QED} \text{ calculable QCD- and QED-radiative corrections}
\]

\[
\frac{\delta_1}{m^2} \text{ nonperturbative}
\]

\[
1 - 8 \rho^2 z(w) + (53 \rho^2 - 15) z(w)^2 - (231 \rho^2 - 91) z(w)^3
\]

with:

\[
\rho^2 \text{ a free parameter.}
\]
Determination of $|V_{cb}|$

- But: phase space vanishes at $w = 1$
  - more difficult to compare to experiment.
  - extrapolation of experimental data to $w = 1$ is done.
- It is important to know the shape of the form factor.

Certain parametrizations can be derived from fundamental principles. 
A widely used one is: [Caprini, Lellouch, Neubert '97]

$$h_{A_1}(w) = \eta_A \eta_{QED} \left( 1 + \frac{\delta_1}{m^2} + \ldots \right) \cdot \left( 1 - 8 \rho^2 z(w) + (53 \rho^2 - 15) z(w)^2 - (231 \rho^2 - 91) z(w)^3 \right)$$

with:

- $\eta_A, \eta_{QED}$ calculable QCD- and QED-radiative corrections
- $\delta_1/m^2$ nonperturbative $1/m^2$-corrections estimated from HQET and inclusive decays
- $z(w) = \left( \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}} \right)$
- $\rho^2$ a free parameter.
Determination of $|V_{cb}|$

- As alternative approaches, the form factor is also calculated in lattice QCD and with QCD sum rules.

- From experimental results, one gets:
  
  - $B \rightarrow D^*$ decays
    
    $$|V_{cb}| = (38.6 \pm 0.9_{\text{exp}} \pm 1.0_{\text{theo}}) \cdot 10^{-3}$$
  
  - $B \rightarrow D$ decays
    
    $$|V_{cb}| = (39.4 \pm 4.2_{\text{exp}} \pm 1.3_{\text{theo}}) \cdot 10^{-3}$$
  
  - Most precise from $B \rightarrow D^*$
    
    $$|V_{cb}|_{\text{excl}} = (38.6 \pm 1.3) \cdot 10^{-3}$$
Good determination of $|V_{cb}|_{excl}$ with the help of heavy quark symmetry.

As it has been shown, the shape (esp. the slope) of the form factor is important for the evaluation.

Lattice QCD and heavy quark symmetry only relyable in the region around $w = 1$. 
Good determination of $|V_{cb}|_{excl}$ with the help of heavy quark symmetry.

As it has been shown, the shape (esp. the slope) of the form factor is important for the evaluation.

Lattice QCD and heavy quark symmetry only relyable in the region around $w = 1$.

The QCD sum rule approach allows the calculation in other kinematical regions and determination of the shape.

The light cone sum rule calculation done in this work gives the form factor in the region around $w = w_{max}$. 
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Method of QCD Sum rules

- Nonperturbative method developed by Shifman, Vainshtein and Zakharov ("SVZ sum rules") in 1979
- Widely used since then for treatment of nonperturbative hadron physics.
- Main principle:
  Usage of the analytical structure of QCD correlation functions, to relate nonperturbative quantities to calculable and/or observable quantities.
Method of QCD Sum rules

- Nonperturbative method developed by Shifman, Vainshtein and Zakharov („SVZ sum rules“) in 1979
- Widely used since then for treatment of nonperturbative hadron physics.

**Main principle:**
Usage of the analytical structure of QCD correlation functions, to relate nonperturbative quantities to calculable and/or observable quantities.

- Method with in principle good, but in the end limited accuracy (as perturbation series is).
- Today lattice QCD gets competitive results (thanks to modern computers and more elaborate theoretical methods).
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Calculation of the B-Meson decay constant

- Starting-point of QCD sum rules:
  
  Consideration of a correlation function.

\[
m_B \langle 0 | \bar{q} \gamma^\mu \gamma^5 b | B(q) \rangle = m_B^2 f_B \langle 0 | \bar{q} \gamma^\mu \gamma^5 b | B(q) \rangle = i q^\mu f_B
\]
Calculation of the B-Meson decay constant

- Starting-point of QCD sum rules:
  
  Consideration of a correlation function.

- Example: Calculation of the B-Meson decay constant $f_B$ defined by:

  \[
  m_b \langle 0 | \bar{q} i \gamma_5 b | B(q) \rangle = m_B^2 f_B \\
  \langle 0 | \bar{q} \gamma_\mu \gamma_5 b | B(q) \rangle = i q_\mu f_B
  \]

- Here we consider the (two-point) correlation function:

\[
\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 \mid T \{ \bar{q}(x) i \gamma_5 b(x), \bar{b}(0) i \gamma_5 q(0) \} \mid 0 \rangle
\]
Now this can be evaluated in two different „languages“:

- Perturbative calculation
- Hadronic representation
Quark-hadron duality

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- Perturbative calculation
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This refers to the quark-hadron duality:

Each hadronic quantity (like a correlation function) is determined by quark interactions, but can only be seen from outside in the different Hilbert-space of hadrons.

*These descriptions have to be equal.*
Now this can be evaluated in two different „languages“:

- Perturbative calculation
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Each hadronic quantity (like a correlation function) is determined by quark interactions, but can only be seen from outside in the different Hilbert-space of hadrons.

*These descriptions have to be equal.*

- In practice this is limited by the truncation of the perturbative series (as well as general problems with its large order behaviour...).

- But a certain estimate can be made.
  → That will be used below: „semi-local quark-hadron duality“

→ More to come on the next slides...
Hadronic representation

Consider the correlation function:

\[ \Pi(q^2) = i \int d^4x \ e^{i q x} \langle 0 \mid T \{ \bar{q}(x) \ i \gamma_5 \ b(x), \ \bar{b}(0) \ i \gamma_5 \ q(0) \} \mid 0 \rangle \]
Hadronic representation

Consider the correlation function:

$$\Pi(q^2) = i \int d^4x \: e^{i q \cdot x} \langle 0 | T \{ \bar{q}(x) \: i \gamma_5 \: b(x), \: \bar{b}(0) \: i \gamma_5 \: q(0) \} | 0 \rangle$$

and put in a complete sum over hadronic intermediate states

$$1 = \sum_h |h\rangle \langle h| :$$

$$\Pi(q^2) = \sum_h i \int d^4x \: e^{i q \cdot x} \langle 0 | \bar{q}(x) \: i \gamma_5 \: b(x) | h \rangle \langle h | \bar{b}(0) \: i \gamma_5 \: q(0) | 0 \rangle = \cdots = \int_{m_B^2}^{\infty} ds \frac{\rho(s)}{s - q^2}$$
Consider the correlation function:

$$\Pi(q^2) = i \int d^4x \, e^{iqx} \langle 0 \mid T \{ \bar{q}(x) i\gamma_5 b(x), \bar{b}(0) i\gamma_5 q(0) \} \mid 0 \rangle$$

and put in a complete sum over hadronic intermediate states $1 = \sum_h |h\rangle\langle h|:$

$$\Pi(q^2) = \sum_h i \int d^4x \, e^{iqx} \langle 0 \mid \bar{q}(x) i\gamma_5 b(x) | h \rangle \langle h | \bar{b}(0) i\gamma_5 q(0) \mid 0 \rangle = \cdots = \int_{m_B^2}^{\infty} ds \, \frac{\rho(s)}{s - q^2}$$

Use the definition of the decay constant $f_B$ to write the first term separately:

$$\Pi^{(hadr)}(q^2) = \frac{m_B^4 f_B^2}{m_b^2(m_B^2 - q^2)} + \int_{s_0^h}^{\infty} ds \, \frac{\rho(s)}{s - q^2}$$

→ Hadronic spectral density:

This is the hadronic dispersion relation.
The correlation function can be evaluated perturbatively, by calculating the corresponding loop diagrams:

\[ \Pi^{(\text{pert})}(q^2) = \frac{3}{8\pi^2} \left( B_0(0,m_b^2,m_b^2)m_b^2 + m_b^2 - B_0(q^2,0,m_b^2)(m_b^2 - q^2) \right) + \mathcal{O}(\alpha_s) \]
The correlation function can be evaluated perturbatively, by calculating the corresponding loop diagrams:

\[ \Pi^{(\text{pert})}(q^2) = \frac{3}{8\pi^2} \left( B_0(0,m^2_b,m^2_b)m^2_b + m^2_b - B_0(q^2,0,m^2_b)(m^2_b - q^2) \right) + \mathcal{O}(\alpha_s) \]

But this is not the whole part of the calculation. Since we're in QCD, we have to take into account nonperturbative effects occurring from small internal loop momenta.

The above part is the first order of an operator-product-expansion, which takes into account these contributions in form of the so-called vacuum condensates.
OPE and vacuum condensates

\[ \Pi(q^2) = \sum_d C_d(q^2) \langle 0 \mid O_d \mid 0 \rangle \]

\[ = \Pi^{(\text{pert})}(q^2) + \frac{m_b}{q^2 - m_b^2} \langle \bar{q}q \rangle + \ldots \]

with:

- \( O_d \): Operators with the quantum numbers of the vacuum and dimension \( d \)
- \( \langle 0 \mid O_d \mid 0 \rangle \): The corresponding condensates, which are nonperturbative quantities, derivable by other sum rules and methods
- \( C_d(q^2) \): Perturbatively calculable Wilson-coefficients
- \( \langle \bar{q}q \rangle \): The quark-condensate (next-leading term)

The first terms in the OPE are:
Calculation of the B-Meson decay constant

One can derive a dispersion form for $\Pi^{(pert)}(q^2)$ and show:

$$\Pi^{(pert)}(q^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} ds \frac{\Im \Pi^{(pert)}(s)}{s - q^2}$$
Calculation of the B-Meson decay constant

One can derive a dispersion form for $\Pi^{(\text{pert})}(q^2)$ and show:

$$\Pi^{(\text{pert})}(q^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} ds \frac{\Im m(\Pi^{(\text{pert})}(s))}{s - q^2}$$

Now we can in principle compare the hadronic part and the perturbative part, to get:

$$\frac{m_B^4 f_B^2}{m_B^2(m_B^2 - q^2)} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - q^2} = \frac{1}{\pi} \int_{m_b^2}^{\infty} ds \frac{\Im m(\Pi^{(\text{pert})}(s))}{s - q^2} + \frac{m_b}{q^2 - m_b^2} \langle \bar{q}q \rangle + \ldots$$
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Here we make use of the „semi-local“ quark-hadron duality:

$$\int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - q^2} \simeq \frac{1}{\pi} \int_{s_0^B}^{\infty} ds \frac{\Im m(\Pi^{(pert)}(s))}{s - q^2} + \ldots$$

→ New „method-inherent“ parameter $s_0$

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Calculation of the B-Meson decay constant

We get the result:

$$\frac{m_B^4 f_B^2}{m_b^2 (m_B^2 - q^2)} \simeq \frac{1}{\pi} \int_{m_b^2}^{s_B} ds \frac{\Im m(\Pi^{(pert)}(s))}{s - q^2} + \frac{m_b}{q^2 - m_b^2} \langle \bar{q}q \rangle$$
Calculation of the B-Meson decay constant

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\[
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\]

To improve the convergence of the series and suppress higher order condensates, we apply a Borel transformation:

\[
\Pi(M^2) = B_{M^2} \Pi(q^2) = \lim_{-q^2, n \to \infty, -\frac{q^2}{n} = M^2} \left( \frac{(-q^2)^{n+1}}{n!} \left( \frac{d}{dq^2} \right)^n \Pi(q^2) \right)
\]

where the momentum \( q^2 \) is transformed to the ("method-inherent") Borel-parameter \( M^2 \).
Calculation of the B-Meson decay constant

In the end, we get the final sum rule:

\[
f_B^2 = \frac{3m_b^2}{8\pi^2 m_B^4} \int_{m_b^2}^{s_B} ds \frac{(s - m_b^2)^2}{s} e^{\frac{m_B^2 - s}{M^2}} - \frac{m_b^2}{m_B^4} (m_b\langle \bar{q}q\rangle) e^{\frac{m_B^2 - m_b^2}{M^2}}
\]
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In dependence of \( M^2 \) one gets with the corresponding input values:

We get an estimate of \( f_B \simeq 150 \text{MeV} \).

More elaborate studies calculate:

\[ f_B = (210 \pm 19) \text{MeV} \]

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Calculation of the B-Meson decay constant

In the end, we get the final sum rule:

$$f_B^2 = \frac{3 m_b^2}{8 \pi^2 m_B^4} \int_{m_b^2}^B ds \left( \frac{s - m_b^2}{s} \right)^2 e^\frac{m_B^2 - s}{M^2} - \frac{m_b^2}{m_B^4} (m_b \langle \bar{q}q \rangle) e^\frac{m_B^2 - m_b^2}{M^2}$$

In dependence of $M^2$ one gets with the corresponding input values:

We get an estimate of $f_B \simeq 150 \, \text{MeV}$.

More elaborate studies calculate: [Jamin, Lange 2001]

$$f_B = (210 \pm 19) \, \text{MeV}$$
Here have a method to „translate“ perturbative calculations and universal nonperturbative parameters (condensates) into hadronic observables.
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Here have a method to „translate“ perturbative calculations and universal nonperturbative parameters (condensates) into hadronic observables. This is the main method of the QCD sum rule approach. All other calculations go the same way.

Now the method shall be applied to $B \to D(\ast)$ decays in the framework of light cone sum rules (LCSR). This work is based on the corresponding approach to the $B \to \pi$ decay, done by A. Khodjamirian, Th. Mannel and N. Offen.
1. Theory of $B \to D^{(*)}$ decays
   - Introduction: Determination of $|V_{cb}|$
   - Use of Heavy Quark Symmetry

2. QCD Sum rules
   - Basic idea of QCD sum rules
   - Light cone sum rules
   - Distribution amplitudes

3. Results and Discussion
   - Numerical results
   - Summary and Outlook
Three point sum rules in principle can be used to calculate meson decay form factors:

Consider a correlator of three currents.

Nonperturbative contributions in form of vacuum condensates.

Disadvantage/uncertainty: Difficult theoretical treatment

- Double dispersion relation / hadronic spectral density
- Problems with applicability of OPE
- ...
Method of light cone sum rules

Description of heavy-meson decays:

- 90’s: Three-point sum rules [Ball, Colangelo, Ovchinnikov, Slobodenyuk, Radyushkin et. al.]
- Today more advanced method: light cone sum rules
Method of light cone sum rules

Description of heavy-meson decays:

- 90’s: Three-point sum rules [Ball, Colangelo, Ovchinnikov, Slobodenyuk, Radyushkin et. al.]
- Today more advanced method: light cone sum rules

Considered correlation function

\[
F_{ab}(p, q) = i \int d^4 x \ e^{ipx} \langle 0 | T \{\bar{q}(x)\Gamma_a c(x), \bar{c}(0)\Gamma_b b(0)\} | B \rangle
\]

Christoph Klein (Univ. Siegen)
Hadronic representation

Choose e.g. $\Gamma_a = i\gamma_5$, $\Gamma_b = \gamma_\mu$:

$$F_\mu(p,q) = i \int d^4x \ e^{ipx} \langle 0 | T \{\bar{q}(x)i\gamma_5 c(x), \bar{c}(0)\gamma_\mu b(0)\} | B \rangle$$
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→ Put in a complete sum over hadronic intermediate states $1 = \sum_h |h\rangle\langle h|$: 

$$F_\mu(p,q) = \langle 0 | \bar{q}i\gamma_5 c | D \rangle \langle D | \bar{c}\gamma_\mu b | B \rangle \frac{m_D^2 - p^2}{s - p^2} + \int_{s_0}^{\infty} ds \frac{\rho(s)}{s - p^2}$$

→ Hadronic spectral density
Hadronic representation

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→ Put in a complete sum over hadronic intermediate states $1 = \sum_h |h\rangle \langle h|$

$$F_\mu(p,q) = \frac{\langle 0 | \bar{q} i\gamma_5 c | D \rangle \langle D | \bar{c} \gamma_\mu b | B \rangle}{m_D^2 - p^2} + \int_{s_0}^{\infty} ds \frac{\rho(s)}{s - p^2}$$

→ Hadronic spectral density

→ Access to form factor-definition:

$$F_\mu(p,q) \sim f_D \cdot \left( \frac{(2p_\mu + q_\mu) f^+(q^2) + q_\mu f^-(q^2)}{m_D^2 - p^2} \right) + \cdots$$
Calculation of the correlation function

The correlation function can by evaluated using the operator-product-expansion (OPE):

\[ F_{ab}(p,q) = i \int d^4x \ e^{ipx} \langle 0 | T \{ \bar{q}(x) \Gamma_a c(x), \bar{c}(0) \Gamma_b b(0) \} | B \rangle \]
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\]

- B-meson: Transition to HQET: \(|B(p + q) > \rightarrow |B_v > + \mathcal{O}(1/m_b), \ b(0) \rightarrow h_v(0) + \mathcal{O}(1/m_b)\)
  \rightarrow m_b\text{-dependence is neglected, but later comes in with kinematic } m_B\text{-terms}
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- One can prove light-cone-dominance: Only contributions from \( x^2 \approx 0 \)

- Contraction of \( c(x) \bar{c}(0) \) to the free quark-propagator
  (OPE: "light-cone-expansion" for \( q^2, p^2 \ll m_b^2 \))

  \( \rightarrow \) The last is only valid for \( q^2 \) far enough from \( q_{max}^2 = (m_B - m_D)^2 \), i.e. \( w \) far enough from \( w = 1 \).
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- One can prove light-cone-dominance: Only contributions from \(x^2 \approx 0\)

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  \(w\) far enough from \(w = 1\).

\[ F_{ab}(p,q) = i \int d^4x \int \frac{d^4k}{(2\pi)^4} \ e^{i(p-k)x} \left[ \Gamma_a \left( \frac{k + m_c}{k^2 - m_c^2} \right) \Gamma_b \right]_{\beta \alpha} \langle 0 | T \{ \bar{q}_\alpha(x) h_\nu \beta(0) \} | B_\nu \rangle \]
The residual matrix element is the nonperturbative quantity.

It is parametrized by two B-meson light-cone distribution amplitudes $\phi_+(\omega), \phi_-(\omega)$:
Perturbative calculation

\[ F_{ab}(p,q) = i \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(p-k)x} \left[ \Gamma_a \left( \frac{k + m_c}{k^2 - m_c^2} \right) \Gamma_b \right] \langle 0 | T \{ \bar{q}_\alpha(x) h_{\nu\beta}(0) \} | B_v \rangle \]

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\[ \langle 0 \mid \bar{q}_\alpha(x) h_{\nu\beta}(0) \mid B_v \rangle = - \frac{i f_B m_B}{4} \int_0^\infty d\omega e^{-i \omega v \cdot x} \left[ (1 + \gamma) \left\{ \frac{\phi_+ (\omega)}{2v \cdot x} - \frac{\phi_+ (\omega) - \phi_- (\omega)}{2v \cdot x} \right\} \gamma^5 \right]_{\beta \alpha} \]

- These are dependent of a dimensionful variable \( \omega \):
  Projection of light-quark momentum onto the light-cone.
Perturbative calculation

\[ F_{ab}(p,q) = i \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(p-k)x} \left[ \Gamma_a \left( \frac{k + m_c}{k^2 - m_c^2} \right) \Gamma_b \right]_{\beta\alpha} \langle 0 | T \{ \bar{q}_\alpha(x) h_{v\beta}(0) \} | B_v \rangle \]

- The residual matrix element is the nonperturbative quantity.
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- These are dependent of a dimensionful variable \( \omega \):
  Projection of light-quark momentum onto the light-cone.

\[ \Rightarrow F_{ab}(p,q) = f \left[ \phi_+(\omega), \phi_-(\omega), p, q \right] \]
Correlation function:

\[ F_{ab}(p,q) = i \int d^4x \ e^{ipx} \langle 0 \mid T \{ \bar{q}(x)\Gamma_a c(x), \bar{c}(0)\Gamma_b b(0) \} \mid B \rangle \]

\[ F_{ab}(p,q) = \frac{\langle 0 \mid \bar{q}\Gamma_a c \mid D \rangle \langle D \mid \bar{c}\Gamma_b b \mid B \rangle}{m_D^2 - p^2} + \ldots \]

\[ \sim f_+, f_- \]

\[ \rightarrow \text{quark-hadron-duality (parameter } s_0) \]

\[ \rightarrow \text{Borel-transformation (parameter } M^2) \]

\[ \rightarrow \text{OPE & light-cone-expansion} \]

\[ F_{ab}(p,q) \sim \langle 0 \mid \bar{q}(x) b(0) \mid B \rangle \]

\[ \sim \phi_+, \phi_- \]

\[ \Rightarrow \text{Sum rule:} \]

\[ f_+(q^2), f_-(q^2) = f [\phi_+(\omega), \phi_-(\omega), q^2, \ldots] \]
Outline

1. Theory of $B \to D^{(*)}$ decays
   - Introduction: Determination of $|V_{cb}|$
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The important **nonperturbative** input parameters are the light-cone distribution amplitudes (DAs) $\phi_+ (\omega), \phi_- (\omega)$.

Generally:
Light-cone distribution amplitudes are important (universal) quantities in the description of heavy meson decays and describe the momentum distribution of the quarks/gluons in the meson. (Here in the 2-particle Fock state.)

Central role in factorization theorems of hadronic decays.
The important nonperturbative input parameters are the light-cone distribution amplitudes (DAs) $\phi_+(\omega), \phi_-(\omega)$.

Generally:
Light-cone distribution amplitudes are important (universal) quantities in the description of heavy meson decays and describe the momentum distribution of the quarks/gluons in the meson. (Here in the 2-particle Fock state.)

Central role in factorization theorems of hadronic decays.

In principle one has access to the DAs with two-point sum rules.

But: Theoretical problems in the condensate expansion.
   → No direct calculation of the DAs, but constraints and implications.
   → Use of models.
Models for distribution amplitudes

The models are suggested under some general aspects:

- Consistent behaviour of the momentum distribution with respect to QCD equations of motion.
- Right behaviour under renormalization.

Example:
The light meson distribution amplitudes (Pion DAs) can be modelled as an expansion in orthogonal Gegenbauer-polynomials.

For example the leading DA of the corresponding OPE (twist expansion) has the form:

$$ \phi_{\pi}(u, \mu) = 6u(1-u) + X_{n=2,4,...} a_n(\mu) C_{3/2}^n(u-(1-u)) $$

The coefficients $a_n(\mu)$ can be determined from two-point sum rules.
The models are suggested under some general aspects:

- Consistent behaviour of the momentum distribution with respect to QCD equations of motion.
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Example:

The **light meson distribution amplitudes** (Pion DAs) can be modelled as an expansion in orthogonal Gegenbauer-polynomials.

For example the leading DA of the corresponding OPE (twist expansion) has the form:

\[
\phi_\pi(u, \mu) = 6u(1 - u) \left( 1 + \sum_{n=2,4,...} a_n(\mu) C_n^{3/2} (u - (1 - u)) \right)
\]

The coefficients \(a_n(\mu)\) can be determined from two-point sum rules.
**LCSR with light meson DAs**

- The pion DAs are very well studied and known in a good accuracy.
- The $B \to \pi$ - form factors have been studied extensively with these DAs and light cone sum rules.

**Correlation functions**

Contributions by A. Khodjamirian, N. Offen, R. Rückl, P. Ball and many others...

→ Determination of $|V_{ub}|$ from exclusive $B \to \pi$ decays.
The B-meson DAs cannot be expanded in that way ...

But: Two-point sum rules suggest a behaviour:

\[ \phi_+(\omega \to 0) \sim \omega, \phi_-(\omega \to 0) \to \text{const} \]
The B-meson DAs cannot be expanded in that way ...

But: Two-point sum rules suggest a behaviour:

\[ \phi_+ (\omega \to 0) \sim \omega, \phi_- (\omega \to 0) \longrightarrow \text{const} \]

We use an exponential model: [Grozin and Neubert (1997)]

\[
\phi_+ (\omega) = \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}} \\
\phi_- (\omega) = \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}}
\]

\[
\frac{1}{\lambda_B} = \int_0^\infty d\omega \frac{\phi_+(\omega)}{\omega} \\
\omega_0 = \lambda_B = (460 \pm 110) \text{ MeV}
\]

[ Braun, Ivanov, Korchemsky (2003) ]

Yet the knowledge about these DAs is much less precise than the light meson DAs.
Additional: Consideration of the 3-particle-contribution

\[
\langle 0 | \bar{q}_{2\alpha}(x) G_{\lambda \rho}(ux) h_{\nu \beta}(0) | \bar{B}_v \rangle = \frac{f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi e^{-i(\omega + u\xi) \cdot x} \left[ (1 + \gamma_5) \left\{ (\nu_\lambda \gamma_\rho - \nu_\rho \gamma_\lambda)(\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)) - i\sigma_\lambda \rho \Psi_V(\omega, \xi) \right. \right.
\]

\[
- \left( \frac{x_\lambda \nu_\rho - x_\rho \nu_\lambda}{\nu \cdot x} \right) X_A(\omega, \xi) - \left( \frac{x_\lambda \nu_\rho - x_\rho \nu_\lambda}{\nu \cdot x} \right) Y_A(\omega, \xi) \right\} \gamma_5 \right]_{\beta \alpha}
\]

\[\rightarrow}\]
Exponential model: [A. Khodjamirian, Th. Mannel, N. Offen (2007)]
B-meson-3-particle-distribution amplitudes

Additional: Consideration of the 3-particle-contribution

\[ \langle 0 \mid \bar{q}_{2\alpha}(x) \, G_{\lambda\rho}(ux) \, h_{\nu\beta}(0) \mid \bar{B}_v \rangle \]

\[ = \frac{f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi \, e^{-i(\omega + u\xi) \cdot x} \left( 1 + \gamma^5 \right) \]

\[ \left\{ (v_\lambda \gamma_{\rho} - v_\rho \gamma_{\lambda})(\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)) - i\sigma_{\lambda\rho} \Psi_V(\omega, \xi) \right\} \gamma^5_{\beta\alpha} \]

\[ \Psi_A(\omega, \xi) = \Psi_V(\omega, \xi) = \frac{\lambda E^2}{6\omega_0^4} \xi^2 e^{-\frac{(\omega + \xi)}{\omega_0}} \]

\[ X_A(\omega, \xi) = \frac{\lambda E^2}{6\omega_0^4} \xi(2\omega - \xi)e^{-\frac{(\omega + \xi)}{\omega_0}} \]

\[ Y_A(\omega, \xi) = -\frac{\lambda E^2}{24\omega_0^4} \xi(7\omega_0 - 13\omega + 3\xi)e^{-\frac{(\omega + \xi)}{\omega_0}} \]

Applying the same method as for 2-particle-DAs:

→ Exponential model: [A. Khodjamirian, Th. Mannel, N. Offen (2007)]

\[ B \rightarrow D^{(*)} \text{ formf. from QCD sum rules} \]
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Results for the form factors

Example: sum rule for $f_+(q^2)$

\[
\begin{align*}
f_+(q^2) &= \frac{f_B m_c}{2 f_D m_D^2} \int_{0}^{\omega_0(q^2,s_0)} d\omega \exp \left( -s \left( \omega, q^2 \right) + m_D^2 \right) \\
&\quad \times \left\{ \frac{(m_B - \omega + m_c) \phi_+ (\omega) - \Phi_+ (\omega)}{(1 - \omega/m_B)} \right. \\
&\quad \left. - \frac{1}{(m_B - \omega)^2 + m_c^2 - q^2} \left( \frac{2 m_B m_c (m_B - \omega) (m_B - \omega + m_c)}{(m_B - \omega)^2 + m_c^2 - q^2} \Phi_+ (\omega) \\
&\quad + m_B m_c \left( (m_B - \omega + m_c) \left( \phi_+ (\omega) - \phi_- (\omega) \right) - \Phi_+ (\omega) \right) \right\} \\
&\quad + \left( \Delta f_+(q^2) \right)_{\text{3-particle}}
\end{align*}
\]

\[
\begin{align*}
s(\omega, q^2) &= \omega m_B + \frac{m_B m_c^2 - \omega q^2}{m_B - \omega} \\
\Phi_+ (\omega) &= \int_{0}^{\infty} d\tau \left( \phi_+ (\tau) - \phi_- (\tau) \right) \\
\omega_0(q^2,s_0) &= \frac{1}{2m_B} \left( m_B^2 - q^2 + s_0 - \sqrt{4 \left( m_c^2 - s_0 \right) m_B^2 + \left( m_B^2 - q^2 + s_0 \right)^2} \right)
\end{align*}
\]
Results

\( B \to D \)-form factors (straight) and only 2-particle-contribution (dashed)

\[ f_+(q^2) \] with uncertainty (left) and \( M^2 \)-dependence (right)
Results

$B \to D^*$-form factors (straight line) and only 2-particle-contribution (dashed)

$A_1(q^2)$ with uncertainty (left) and $M^2$-dependence (right)
Comparison with experiment (BABAR ’08)

Definition of the form factors in HQET:

\[
\begin{aligned}
\langle D(v') | V^{\mu} | \bar{B}(v) \rangle &= h_+(w)(v + v')^{\mu} + h_-(w)(v - v')^{\mu} \\
\langle D^*(v', \epsilon) | A^{\mu} | \bar{B}(v) \rangle &= -i \epsilon^{* \mu} (w + 1) h_{A1}(w) + i (\epsilon^* \cdot q) v^{\mu} h_{A2}(w) + i (\epsilon^* \cdot q) v'^{\mu} h_{A3}(w)
\end{aligned}
\]

\[
\begin{aligned}
(w = v \cdot v' = \frac{m_B^2 + m_{D(*)}^2 - q^2}{2 m_B m_{D(*)}})
\end{aligned}
\]
Comparison with experiment  \((\text{BABAR '08})\)

- **Definition of the form factors in HQET:**

  \[
  \frac{\langle D(v')|V^{\mu} |\bar{B}(v)\rangle}{\sqrt{m_B m_D}} = h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu
  \]
  \[
  \frac{\langle D^*(v',\epsilon)|V^{\mu} |\bar{B}(v)\rangle}{\sqrt{m_B m_D^*}} = h_V(w)\epsilon^{\nu\alpha\beta} \epsilon^{*}_{\nu'} v^\alpha v'^\beta
  \]
  \[
  \frac{\langle D^*(v',\epsilon)|A^{\mu} |\bar{B}(v)\rangle}{\sqrt{m_B m_D^*}} = -i\epsilon^{*\mu}(w + 1)h_{A1}(w) + i(\epsilon^{*}\cdot q) v^\mu h_{A2}(w) + i(\epsilon^{*}\cdot q) v'^\mu h_{A3}(w)
  \]

- **HQET-prediction for** \(m_b, m_c \to \infty\):

  \[
  h_+(w) = h_V(w) = h_{A1}(w) = h_{A3}(w) = \xi(w) \quad \rightarrow \text{Isgur-Wise-function}
  \]
  \[
  h_-(w) = h_{A2}(w) = 0
  \]
Results

**Comparison with experiment**  \( (\text{BABAR '08}) \)

- **Definition of the form factors in HQET:**

\[
\langle D'(v') | V^{\mu} | \bar{B}(v) \rangle \!
\frac{1}{\sqrt{m_B m_D}} = h_+(w)(v + v')^{\mu} + h_-(w)(v - v')^{\mu}
\]

\[
\langle D^*(v', \epsilon) | V^{\mu} | \bar{B}(v) \rangle \!
\frac{1}{\sqrt{m_B m_D^*}} = h_V(w) \epsilon^\mu \epsilon^{\nu \alpha \beta} e^\nu_{\nu'} e^\gamma_{\gamma'}
\]

\[
\langle D^*(v', \epsilon) | A^{\mu} | \bar{B}(v) \rangle \!
\frac{1}{\sqrt{m_B m_D^*}} = -i \epsilon^\mu (w + 1) h_A_1(w) + i(\epsilon^\mu \cdot q) v^{\mu} h_A_2(w) + i(\epsilon^\mu \cdot q) v'^\mu h_A_3(w)
\]

- **HQET-prediction for \( m_b, m_c \to \infty \):**

\[
h_+(w) = h_V(w) = h_A_1(w) = h_A_3(w) = \xi(w) \quad \rightarrow \text{Isgur-Wise function}
\]

\[
h_-(w) = h_A_2(w) = 0
\]

- **Experimentally measurable:**

\[
G(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w)
\]

\[
R_1(w) = \frac{h_V(w)}{h_A_1(w)}
\]

\[
R_2(w) = \frac{m_D^*}{m_B} h_A_2(w) + h_A_3(w) \quad \text{h_{A_1}(w)}
\]
Comparison with experiment (BABAR ’08)

Form factors $G(w), h_{A_1}(w)$  

[Sum rules: $(w = 1.3 - w_{\text{max}})$, exp.: $(w = 1 - w_{\text{max}})$]

Sum rules only applicable for small momentum transfer: $w \gg 1 \rightarrow w \simeq 1.3 - w_{\text{max}}$

Fit to experimental data with the parametrization:  
[Caprini, Lellouch, Neubert ’97]

\[
\begin{align*}
\quad h_{A_1}(w) &= h_{A_1}(1) \left(1 - 8 \rho^2 z(w) + (53 \rho^2 - 15) z(w)^2 - (231 \rho^2 - 91) z(w)^3\right) \\
G(w) &= G(1) \left(1 - 8 \rho^2 z(w) + (51 \rho^2 - 10) z(w)^2 - (252 \rho^2 - 84) z(w)^3\right)
\end{align*}
\]

\[z(w) = \left(\frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}\right)\]

→ Normalization $h_{A_1}(1), G(1)$ from Lattice-QCD
Comparison with experiment (BABAR ’08)

Ratios $R_1(w), R_2(w)$

$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2$

$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2$

→ In the ratios the normalizations cancel: better prediction
Limit of infinitely heavy quarks

→ Taking the limit: \( m_c, m_b \rightarrow \infty, \kappa := \frac{m_c}{m_b} = const \) for the sum rules:

\[
\begin{align*}
  h_+(w) &= h_V(w) = h_{A_1}(w) = h_{A_3}(w) \\
  \xi(w) &= \beta_0/w \int_0^\infty d\rho \ e^{(\frac{\bar{\Lambda} - \rho w}{\tau})} \left[ \frac{1}{2w} \phi_B(\rho) + (1 - \frac{1}{2w}) \phi^+(\rho) \right] \\
  h_-(w) &= h_{A_2}(w) = 0
\end{align*}
\]

Scaling of the parameter: \( m_B = m_b + \bar{\Lambda}, \ m_D = m_c + \bar{\Lambda}, \ s_0^D = m_c^2 + 2m_c\beta_0, \ M^2 = 2m_c\tau \)
Limit of infinitely heavy quarks

→ Taking the limit: \( m_c, m_b \rightarrow \infty, \kappa := \frac{m_c}{m_b} = \text{const} \) for the sum rules:

\[
\begin{align*}
  h_+(w) &= h_V(w) = h_{A_1}(w) = h_{A_3}(w) \\
  : &= \xi(w) = \int_0^{\beta_0/w} d\rho \ e^{\left(\frac{\bar{\Lambda} - \rho}{\tau}\right)} \left[ \frac{1}{2w} \phi_B^-(\rho) + \left(1 - \frac{1}{2w}\right) \phi_B^+(\rho) \right] \\
  h_-(w) &= h_{A_2}(w) = 0
\end{align*}
\]

Scaling of the parameter: \( m_B = m_b + \bar{\Lambda}, \ m_D = m_c + \bar{\Lambda}, \ s_0^D = m_c^2 + 2m_c\beta_0, \ M^2 = 2m_c\tau \)

- Independent of the quark mass ratio \( \kappa \)
- Numerical one gets the right value (taking into account the uncertainty):

\[ \xi(1) \approx 1 \]
Theory of $B \to D^{(*)}$ decays
- Introduction: Determination of $|V_{cb}|$
- Use of Heavy Quark Symmetry

QCD Sum rules
- Basic idea of QCD sum rules
- Light cone sum rules
- Distribution amplitudes

Results and Discussion
- Numerical results
- Summary and Outlook
Summary

- Sum rules derived for all $B \to D^{(*)}$-form factors.
  - Only applicable for small momentum transfer $q^2 \ll q^2_{\text{max}}$ bzw. $w \approx (1,3 - w_{\text{max}})$.
  - But this is the region, where HQS and Lattice QCD have difficulties.

- Two- and three-particle-contributions considered.

- Good agreement with experimental data.

- Right behaviour in the limit $m_c, m_b \to \infty$. 

Relative big uncertainty, mainly due to errors in the input quantities: $B$-meson-DAs and decay constants. Yet, in accuracy the result cannot compete with the HQS-determination, but it is an important cross-check and allows more insight into the form factor shape and the heavy quark limit. Further studies of $1/m$-corrections in HQET are possible here.
Summary

- Sum rules derived for all $B \rightarrow D^{(*)}$-form factors.
  Only applicable for small momentum transfer $q^2 \ll q_{\text{max}}^2$ bzw. $w \simeq (1,3-w_{\text{max}})$.
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- Yet, in accuracy the result cannot compete with the HQS-determination, but it is an important cross-check and allows more insight into the form factor shape and the heavy quark limit.

- Further studies of $1/m$-corrections in HQET are possible here.
In the future ...:

- Closer examination of $\frac{1}{m_Q}$-expansion
- Determination of $\frac{|V_{ub}|}{|V_{cb}|}$ using the sum rules $\rightarrow$ smaller uncertainty
- Examination/calculation of $\alpha_s$-corrections (currently at work)

Further project: Evaluation of light cone sum rules with $\pi$-meson-distribution amplitudes for $D \rightarrow \pi$- and $D \rightarrow K$-form factors with A. Khodjamirian and N. Offen

...?
„Ich habe Jahre gebraucht, um sagen zu können, dass alles ganz einfach ist.‘‘

„It took me years to get able to, but now I can say that all this is simple.‘‘

- N. Offen

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