Introduction and Motivation
Application: Endpoint region in $B \rightarrow X_c \ell \bar{\nu}_\ell$
Does it help to get $V_{ub}$?

The Charm Quark
as a
Massive Collinear Quark

Thomas Mannel
(with Heike Boos, Thorsten Feldmann and Ben Pecjak)

Theoretische Physik I
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Charm as a Massive Collinear Quark
SCET in an nutshell

- Special kinematic situation: Light degrees of freedom, which are fast in the rest frame of a heavy decaying object
  - $B \rightarrow \pi \pi$: fast pions
  - $B \rightarrow X_S \gamma$ in the endpoint region: Collimated $X_S$ jet
  - $B \rightarrow X_u \ell \bar{\nu}_\ell$ in the endpoint region

- Light Cone Decomposition:
  light-like vectors $n_+^\mu$ and $n_-^\mu$ with $n_+ n_- = 2$ and

  \[ p^\mu = (n_+ p) \frac{n_+^\mu}{2} + p_\perp^\mu + (n_- p) \frac{n_-^\mu}{2}, \quad p^2 = (n_+ p)(n_- p) + p_\perp^2 \]

- SCET kinematics: One large light cone component

(Bauer, Stewart, Pirjol, Flemming, Rothstein, Beneke, Feldmann, ...)

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Light Cone Decomposition:
light-like vectors $n_\mu^+$ and $n_\mu^-$ with $n_+n_- = 2$ and

$$p_\mu = (n_+p)\frac{n_\mu^+}{2} + p_\perp + (n_-p)\frac{n_\mu^-}{2}, \quad p^2 = (n_+p)(n_-p)+p_\perp^2$$

SCET kinematics: One large light cone component

(Bauer, Stewart, Pirjol, Flemming, Rothstein, Beneke, Feldmann, ...)

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**Power Counting**

- **Hard Momenta:** \((n_+ p, p_\perp, n_- p) \sim m_b (1, 1, 1)\)
- **Collinear Momenta:** \((n_+ p, p_\perp, n_- p) \sim m_b (1, \lambda, \lambda^2)\)
- **Soft Momenta:** \((n_+ p, p_\perp, n_- p) \sim m_b (\lambda^2, \lambda^2, \lambda^2)\)

\[ \lambda = \sqrt{\frac{p_{\text{jet}}^2}{m_b^2}} = \sqrt{\frac{\Lambda}{m_b}} \ll 1 \]

- Construct a Lagrangian to implement this
SCET in a nutshell

Mass Terms in SCET

Power Counting

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\[
p_{\text{coll}}^2 \sim m_b^2 \lambda^2 \quad p_{\text{soft}}^2 \sim m_b^2 \lambda^4
\]

Construct a Lagrangian to implement this
**Introduction and Motivation**

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**SCET in a nutshell**

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**Construct a Lagrangian to implement this**

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$$p^2_{\text{coll}} \sim m_b^2 \lambda^2 \quad p^2_{\text{soft}} \sim m_b^2 \lambda^4$$

Construct a Lagrangian to implement this
Power Counting

- **Hard Momenta:** \((n_+ p, p_\perp, n_- p) \sim m_b(1, 1, 1)\)
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p^2_{\text{coll}} \sim m_b^2 \lambda^2 \quad p^2_{\text{soft}} \sim m_b^2 \lambda^4
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Construct a Lagrangian to implement this
Lagrangian of SCET

Rewrite QCD-Lagrangian:

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}(i\mathcal{D})\psi = \bar{\xi}\left((\text{in}_-\mathcal{D}) + (i\mathcal{D}_\perp)\frac{1}{\text{in}_+\mathcal{D}(i\mathcal{D}_\perp)}\right)\frac{\hat{n}_+}{2}\xi \]

Fields on the light cone:

\[ \xi(x) = \frac{\hat{n}_- - \hat{n}_+}{4}\psi(x) \]

\[ \psi(x) = \left(1 + \frac{1}{(\text{in}_-\mathcal{D})(i\mathcal{D}_\perp)\frac{\hat{n}_+}{2}}\right)\xi(x) \]

Expand this according to the power counting.
Lagrangian of SCET

- **Rewrite QCD-Lagrangian:**
  \[
  \mathcal{L}_{\text{QCD}} = \bar{\psi}(i\slashed{D})\psi = \bar{\xi}
  \left( i\slashed{n} - D \right) + (i\slashed{D}_{\perp}) \frac{1}{in_{+}D}(i\slashed{D}_{\perp}) \right) \frac{\eta_{+}}{2}\xi
  \]

- **Fields on the light cone:**
  \[
  \xi(x) = \frac{\eta_{-} - \eta_{+}}{4} \psi(x)
  \]
  \[
  \psi(x) = \left( 1 + \frac{1}{i\slashed{n} - D}(i\slashed{D}_{\perp}) \frac{\eta_{+}}{2} \right) \xi(x)
  \]

- **Expand this according to the power counting**
Lagrangian of SCET

- Rewrite QCD-Lagrangian:

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}(i\slashed{D})\psi \]

\[ = \bar{\xi}\left((\slashed{in}_- D) + (i\slashed{D}_\perp)\frac{1}{\slashed{in}_+ D}(i\slashed{D}_\perp)\right) \frac{\slashed{n}_+}{2} \xi \]

- Fields on the light cone:

\[ \bar{\xi}(x) = \frac{\slashed{n}_- \slashed{n}_+}{4} \psi(x) \]

\[ \psi(x) = \left(1 + \frac{1}{(\slashed{in}_- D)(i\slashed{D}_\perp)}\frac{\slashed{n}_+}{2}\right)\xi(x) \]

- Expand this according to the power counting
\[(iD_\perp) = (iD_\perp c) + gA_\perp s, \quad \text{(in}_+D) = (\text{in}_+pD_c) + gn_+A_s\]

- **SCET in a nutshell**

**Mass Terms in SCET**

**Power Counting**

1. **Leading term:**

\[
\mathcal{L}_{\text{SCET}} = \bar{\xi} \left( (\text{in}_-D) + (iD_\perp c) \frac{1}{\text{in}_+D_c} (iD_\perp c) \right) \frac{n_+}{2} \xi
\]

2. **Many subleading terms**

(Stewart, Rothstein, Neubert, Beneke, Campanario, M.)

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\( (iD_\perp) = (iD_\perp c) + gA_\perp s \), \( (i n_+ D) = (i n_+ p D_\perp c) + g n_+ A_s \)

- **Multipole Expansion of the soft fields:** (Beneke, Feldmann, Diehl)

\[
\begin{align*}
    x^{\mu} &= (n_+ x) \frac{n_\perp^\mu}{2} + x_\perp^{\mu} + (n_- x) \frac{n_+^\mu}{2} \\
    A_s(x) &= A_s(x_-) + [x_\perp \partial_\perp A_s](x_-) + \cdots
\end{align*}
\]

- **Leading term:**

\[
L_{\text{SCET}} = \bar{\xi} \left( (i n_- D) + (i \not\! D_\perp c) \frac{1}{i n_+ D_\perp c} (i \not\! D_\perp c) \right) \frac{n_+}{2} \xi
\]

- **Many subleading terms** (Stewart, Rothstein, Neubert, Beneke, Campanario, M.)
(iD⊥) = (iD⊥c) + gA⊥s, \quad (in+D) = (in+pDc) + gn+A_s


\[ x^\mu = (n_+x)\frac{n^-_\mu}{2} + x^\mu_\perp + (n_-x)\frac{n^+_\mu}{2} \]

\[ A_s(x) = A_s(x_-) + [x_\perp \partial_\perp A_s](x_-) + \cdots \]

Leading term:

\[ \mathcal{L}_{SCET} = \bar{\xi} \left( (in_-D) + (i\not{D}_\perp c) \frac{1}{in_+D_c} (i\not{D}_\perp c) \right) \frac{1}{2} \xi \]

Many subleading terms (Stewart, Rothstein, Neubert, Beneke, Campanario, M.)
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Power Counting

$(iD_\perp) = (iD_{\perp c}) + gA_{\perp s}$, $(in_+ D) = (in_+ pD_c) + gn_+ A_s$

**Multipole Expansion of the soft fields:**

(Beneke, Feldmann, Diehl)

$$x^{\mu} = (n_+ x) \frac{n^{\mu}_-}{2} + x^{\mu}_\perp + (n_- x) \frac{n^{\mu}_+}{2}$$

$$A_s(x) = A_s(x_-) + [x_\perp \partial_\perp A_s](x_-) + \cdots$$

**Leading term:**

$$\mathcal{L}_{\text{SCET}} = \bar{\xi} \left( (in_- D) + (iP_{\perp c}) \frac{1}{in_+ D_c} (iP_{\perp c}) \right) \frac{\not{n}_+}{2} \xi$$

**Many subleading terms**

(Stewart, Rothstein, Neubert, Beneke, Campanario, M.)
Introducing mass terms is obvious:

\[
\mathcal{L}_{\text{QCD}} = \bar{\psi} (i \not{D} - m_q) \psi \\
= \bar{\xi} \left( (\text{in}_- D) + (i \not{D}_\perp - m_q) \frac{1}{\text{in}_+ D} (i \not{D}_\perp + m_q) \right) \frac{n_+}{2} \xi
\]

The fields remain the same

What is the power counting of the mass?
Introducing mass terms is obvious:

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}(iD - m_q)\psi \]

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\[ \mathcal{L}_{\text{QCD}} = \bar{\psi} (i \not{D} - m_q) \psi \]

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The fields remain the same

What is the power counting of the mass?
Power Counting Including the Mass

1. **Light mass**: $m_q \sim m_b \lambda^2$
   - Leading Lagrangian remains the same
   - Mass appears as a subleading order perturbation

2. **Intermediate mass**: $m_q \sim m_b \lambda$
   - The mass appears in the leading Lagrangian
   - Nontrivial dependence on the Mass

   For the charm we have $m_c^2 \sim \Lambda_{QCD} m_b$

   - The charm quark can be a massive collinear quark

   (Neubert, M.)
Power Counting Including the Mass

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Does it help to get $V_{ub}$?

$B \rightarrow X_c \ell \bar{\nu}_\ell$ in usual $1/m_b$ Expansion

**Endpoint region:** $\rho = m_c^2/m_b^2$, $y = 2E_\ell/m_b$

$$\frac{d\Gamma}{dy} \sim \Theta(1-y-\rho) \left[ 2 + \frac{\lambda_1}{(m_b(1-y))^2} \left( \frac{\rho}{1-y} \right)^2 \left\{ 3 - 4 \frac{\rho}{1-y} \right\} \right]$$

**Using the above power counting:** Order Unity Term!

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Charm as a Massive Collinear Quark
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- Using the above power counting: **Order Unity Term!**

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Charm as a Massive Collinear Quark
B → X_c l \bar{\nu}_l in usual \frac{1}{m_b} Expansion

- Endpoint region: \( \rho = \frac{m_c^2}{m_b^2}, \ y = \frac{2E_l}{m_b} \)

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- Using the above power counting: Order Unity Term!
**SCET for** $B \to X_c \ell \bar{\nu}_\ell$: Tree level

- Endpoint region:
  \[(p + k)^2 \sim \Lambda_{\text{QCD}}^2 m_b\]

- Collinear charm quark propagator

\[
S_c(p + k) = \frac{\not{n} - \not{p}}{2} \frac{i}{n_-(p + k) - \frac{m_c^2}{n+p}} = \frac{\not{n} - \not{u}}{2} \frac{i}{u + n_-k}
\]

- New kinematical variable:
  \[u = (n_-p) - \frac{m_c^2}{n+p}\]

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**Application: Endpoint region in** $B \to X_c \ell \bar{\nu}_\ell$

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**Does it help to get** $V_{ub}$?

**SCET for** $B \to X_c \ell \bar{\nu}_\ell$: QCD Corrections

**SCET for** $B \to X_c \ell \bar{\nu}_\ell$: Tree level
SCET for $B \to X_c \ell \bar{\nu}_\ell$: Tree level

- Endpoint region:
  
  $$(p + k)^2 \sim \Lambda_{QCD} m_b$$

- Collinear charm quark propagator

$$S_c(p + k) = \frac{\hat{n}_-}{2} \frac{i}{n_-(p + k) - \frac{m_c^2}{n+p}} = \frac{\hat{n}_-}{2} \frac{i}{u + n_- k}$$

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- **New kinematical variable:**
  $$u = (n_- p) - \frac{m_c^2}{(n + p)}$$
Factorization Formula

\[ d\Gamma = H \cdot J \otimes S \]

- **H**: Hard Coefficient \( \mu = m_b \)
- **J**: Jet Function
  \[ \mu = \sqrt{\Lambda_{QCD}} m_b \]
- **S**: Soft Function, Shape function \( \mu = \Lambda_{QCD} \)
- Tree level: \( H = 1 \)
- Tree level:
  \[ J = \delta(\omega - u) \]
- Tree level:
  \[ S = f(\omega) \]
- Similar structure to subleading order in \( \Lambda_{QCD}/m_b \)
- More soft functions
**Factorization Formula**

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Factorization Formula

$$d\Gamma = H \cdot J \otimes S$$

- $H$: Hard Coefficient $\mu = m_b$
- $J$: Jet Function $\mu = \sqrt[4]{\Lambda_{QCD}} m_b$
- $S$: Soft Function, Shape function $\mu = \Lambda_{QCD}$

- Tree level: $H = 1$
- Tree level: $J = \delta(\omega - u)$
- Tree level: $S = f(\omega)$

- Similar structure to subleading order in $\Lambda_{QCD}/m_b$
- More soft functions
For $B \to X_u \ell \bar{\nu}_\ell$:

$$\frac{1}{\Gamma_u} \frac{d\Gamma}{d(n_p)} = \left( 1 - \frac{14}{3} \frac{n_p}{m_b} \right) S(n_p) + \frac{s(n_p)}{2m_b} + \frac{1}{3m_b} \left[ t(n_p) + u_a(n_p) - 5u_s(n_p) \right]$$

For $B \to X_c \ell \bar{\nu}_\ell$:

$$\frac{1}{\Gamma_c} \frac{d\Gamma}{d\ell} = \left( 1 - \frac{14}{3} \frac{u}{m_b} - 8 \frac{m_c^2}{m_b^2} \right) S(u) + \frac{s(u)}{2m_b} \left[ -4 \frac{m_c^2}{m_b^2} t_1(u) \right] + \frac{1}{3m_b} \left[ t(u) + u_a(u) - 5u_s(u) \right]$$

(Bosch, Neubert, Paz)

(Boos, Feldmann, M., Pecjak)
Introduction and Motivation

Application: Endpoint region in $B \to X_c \ell \bar{\nu}_\ell$

Does it help to get $V_{ub}$?

$B \to X_c \ell \bar{\nu}_\ell$ in usual $1/m_b$

SCET for $B \to X_c \ell \bar{\nu}_\ell$: Tree level

SCET for $B \to X_c \ell \bar{\nu}_\ell$: QCD Corrections

For $B \to X_u \ell \bar{\nu}_\ell$:

\[
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\]

(Bosch, Neubert, Paz)

For $B \to X_c \ell \bar{\nu}_\ell$:

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\frac{1}{\Gamma_c} \frac{d\Gamma}{d(u)} = \left(1 - \frac{14}{3} \frac{u}{m_b} - 8 \frac{m_c^2}{m_b^2}\right) S(u) + \frac{s(u)}{2m_b} - 4 \frac{m_c^2}{m_b^2} t_1(u) + \frac{1}{3m_b} \left[t(u) + u_a(u) - 5u_s(u)\right]
\]

(Boos, Feldmann, M., Pecjak)

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Charm as a Massive Collinear Quark
SCET for $B \rightarrow X_c \ell \bar{\nu}_\ell$: QCD Corrections

- Compute the diagrams in SCET: leading order in $\lambda$
- $H = 1 + O(\alpha_s)$ is the same as in the massless case
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Jet Function at order $\alpha_s$

$$J(u, n+p) = \delta(u) + \frac{C_F \alpha_s}{4\pi} \left\{ \right.$$ \n\n$$\left( 7 - \pi^2 \right) \delta(u) - 3 \left( \frac{1}{u} \right)^{[\mu^2/n+p]} + \left( \frac{\ln(u n+p/\mu^2)}{u} \right)^{[\mu^2/n+p]}$$ \n
$$+ \Theta(u) \left( \frac{u}{(u + m_c^2/n+p)^2} - \frac{4}{u} \ln \left( 1 + \frac{u n+p}{m_c^2} \right) \right)$$ \n
$$+ \left( 1 + \frac{2\pi^2}{3} \right) \delta(u) - \left( \frac{1}{u} \right)^{[m_c^2/n+p]}$$ \n
$$+ 4 \left( \frac{\ln(u n+p/m_c^2)}{u} \right)^{[m_c^2/n+p]} \left\} \right.$$
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\]
Comparison of $B \rightarrow X_c \ell \bar{\nu}_\ell$ and $B \rightarrow X_u \ell \bar{\nu}_\ell$

- Make use of the correspondence of the two variables

$$u = n_- p - \frac{m_c^2}{n_+ p} \quad \text{in} \quad B \rightarrow X_c \quad \leftrightarrow \quad p_+ = n_- p \quad \text{in} \quad B \rightarrow X_u$$

- Use hadronic variables

$$U = u + \bar{\Lambda} \quad \leftrightarrow \quad P_+ = p_+ + \bar{\Lambda}$$

- Consider a partially integrated spectrum \cite{Bosch, Neubert, Paz}

$$F_c(\Delta) = \frac{1}{\Gamma_c} \int_0^\Delta dU \frac{d\Gamma_c}{dU} = \frac{\Gamma_c(U < \Delta)}{\Gamma_c} = F_u(\Delta) + F_m(\Delta)$$
Comparison of $B \rightarrow X_c \ell \bar{\nu}_\ell$ and $B \rightarrow X_u \ell \bar{\nu}_\ell$

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Both $F_u(\Delta)$ and $F_m(\Delta)$ depend to leading order on the shape function

In $B \rightarrow X_u\ell \bar{\nu}_\ell$ we have to leading order the same shape function as the hadronic input \cite{Bosch, Neubert, Paz}.

Partially integrated rate:

$$F_u(\Delta) = \int_0^\Delta dP_+ \frac{d\Gamma_u}{dP_+}$$

Relate $B \rightarrow X_u\ell \bar{\nu}_\ell$ and $B \rightarrow X_c\ell \bar{\nu}_\ell$:

$$\int_0^\Delta dP_+ \frac{d\Gamma_u}{dP_+} = \frac{|V_{ub}|^2}{|V_{cb}|^2} \int_0^\Delta dU \frac{W(\Delta, U)}{dU} \frac{d\Gamma_c}{dU}$$

This defines a (calculable) weight function.
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Introduction and Motivation

Application: Endpoint region in $B \rightarrow X_c \ell \bar{\nu}_\ell$
Does it help to get $V_{ub}$?

$B \rightarrow X_c \ell \bar{\nu}_\ell$ vs. $B \rightarrow X_u \ell \bar{\nu}_\ell$

- Theoretical $u$ spectrum with model shape function

- Uncertainties
  - Factorization scale: 10 – 15 %
  - Charm mass dependence: < 10 %
  - $\Lambda_{QCD}/m_b$ Contributions $\gtrsim$ 20 %
  - Duality??

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Charm as a Massive Collinear Quark
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Charm as a Massive Collinear Quark
**Theoretical $u$ spectrum with model shape function**

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Simple $D$ and $D^*$ toy model:

$$\frac{1}{\Gamma_c} \frac{d^2 \Gamma_c}{d(n^- P) d(n^+ P)} \sim \delta \left( P^2 - \bar{M}_D^2 \right)$$

thick grey line: theoretical result for $F_u(\Delta)$

solid and dashed lines:
$F_u(\Delta)$ from weight function and toy model for $b \to c$ spectrum in different mass schemes

$\Delta$ must be sufficiently large
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Conclusions

- Charm can be treated as a massive collinear quark
- Power Counting as $m_c^2 \sim \Lambda_{QCD} m_b$
- Possible applications also to exclusive channels
  - $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ at large recoil
  - $B \rightarrow D^{(*)} \pi$
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