Higher Orders in Semileptonic $B$-Decays

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Theory Seminar Siegen
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Introduction

Motivation
Heavy Quark Effective Theory
Inclusive Decay: Calculational Method

Higher Orders

The Non-perturbative Parameters
Corrections in Moments and Spectra

Intrinsic Charm

Relevant Operator
The Effective Theory
Implications of Different Scenarios
Introduction

Higher Orders

Intrinsic Charm

Motivation

Heavy Quark Effective Theory

Inclusive Decay: Calculational Method

Motivation

What Do We Consider?

- Consider inclusive semi-leptonic decay $B \rightarrow X_c \ell \bar{\nu}_\ell$

Why Do We Consider This?

- High statistics in experiment
- Good theoretical control using Heavy Quark Expansion (HQE)

$\Rightarrow$ Precise measurement of $|V_{cb}|$ possible

- Important ingredient for UT: $V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0$
- Determination of $\epsilon_K$ depends on $|V_{cb}|^4$: $\sim 35\%$ of error budget!
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![Diagram showing $B \rightarrow X_c \ell \bar{\nu}_\ell$ decay]

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Cleanest Environment: Semi-leptonic $B$-decays
- Tree level
- Factorized hadronic and leptonic interaction
  ⇒ Two possibilities: exclusive and inclusive decays

Results from [PDG: J. Phys. G 37, 075021 (2010)]

$|V_{cb}|^{\text{excl.}} = (38.7 \pm 1.1) \cdot 10^{-3}$

$|V_{cb}|^{\text{incl.}} = (41.5 \pm 0.7) \cdot 10^{-3}$

⇒ Small Tension
Current Extraction Results

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Non-Perturbative Corrections

- Details later
- $m_b$ is the largest scale in this problem
- Interactions of $b$-quark inside $B$-meson are of order $O(\Lambda_{QCD})$

$\Rightarrow$ Perform an OPE in $\Lambda_{QCD}/m_b$
- Each step: Occurrence of non-perturbative parameters

Perturbative Corrections

- Rate can be written as
  \[ \Gamma = \Gamma_0 + \frac{1}{m_b^2} \sum_i C^i_2(\alpha_s) O^i_5 + \frac{1}{m_b^3} \sum_i C^i_3 O^i_6 + \ldots \]
- Each Wilson Coefficient $C^i_j$ has a power series in $\alpha_s$

$\Rightarrow$ Combined expansion in $\alpha_s$ and $1/m_b$: Heavy Quark Expansion
Theoretical Tools

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Current Status: Theory

\[ \alpha_s^n \]

\[ \frac{1}{m_b^n} \]

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Topics Addressed in this Talk

- Part 2: Corrections to order \( \frac{1}{m_b^4} \) and \( \frac{1}{m_b^5} \)
- Part 3: Subtleties concerning \( m_c \)
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Most Recent Fit Result

| Fit       | $|V_{cb}|$ | $m_b$/GeV | $m_c$/GeV |
|-----------|---------|-----------|-----------|
| RESULT    | 41.91   | 4.566     | 1.101     |
| $\Delta_{\text{exp}}$ | 0.48    | 0.034     | 0.045     |
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| $\Delta \Gamma_{sl}$ | 0.59    |            |           |
| $\Delta_{\text{tot}}$ | 0.85    | 0.055     | 0.078     |

- Experimental errors are competitive with theoretical errors
- General uncertainty due to operators with charm content [hep-ph/0511158]

Used in Fit

- Non-perturbative corrections up to $1/m_b^3$
- Electroweak corrections: Estimated $1 + A_{EW} \approx 1.014$
- Perturbative contributions: Using $\alpha_s$, $\alpha_s^2\beta_0$ and $\alpha_s^3\beta_0^2$ to leading order in $1/m_b$: $A_{\text{pert}} \approx 0.908$

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Basic Ideas of Heavy Quark Effective Theory

Idea

- Describe hadrons with $P_H = M_H v$ containing single heavy quark
- Interaction inside hadron: “Off-shellness” of quark
  \[ p_q = m_q v + k \] with residual momentum $k \sim O(\Lambda_{QCD})$
- Heavy quark spin-flavour symmetry as $m_q \to \infty$

HQET Fields

- Define “light” and “heavy” field using the projectors
  \[ Q(x) = e^{-im_q v \cdot x} \left[ \frac{1 + \frac{v}{2}}{2} e^{im_q v \cdot x} Q(x) + \frac{1 - \frac{v}{2}}{2} e^{im_q v \cdot x} Q(x) \right] \]
- “Integrate out” heavy component $\mathcal{Q}_v(x)$
- It remains the light field $\mathcal{Q}_v(x)$ as a static colour source
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Standard Expansion in HQET

Expand the Lagrangian

- Expand in Terms of $iD/m_Q$:
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- Computations: Propagator depends on residual momentum $iD \rightarrow k$
- Corresponds to expand propagator in $k/m_q$ for higher orders

Comments on Expansion

- Non-perturbative information: Matrix elements of operators
- General form: $\langle H_v | \bar{Q}_v(iD) \ldots (iD) Q_v(x) | H_v \rangle$
- Need to match expansion in $k$ to operators
  - Additional gluon matrix elements to keep track on the ordering of $k$
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Remark on the Following Calculational Method

Outline of the Strategy

- Identify operators in order $1/m_b^n$
- Compute general decomposition of matrix element

Advantages

- Ordering preserved: Straight forward computation
  ⇒ Economical calculation, automatable
- Generalizable to arbitrarily high orders

Disadvantages

- Matrix elements non universal: Using full QCD field
  ⇒ Cannot be used for different heavy quark systems
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Starting Point

Differential Rate

\[ d\Gamma = 16\pi G_F^2 |V_{cb}|^2 W_{\mu\nu} L^{\mu\nu} \, d\phi \]

- \( d\phi \): Phase-space
- \( L^{\mu\nu} \): Leptonic Tensor
- \( W_{\mu\nu} \): Hadronic Tensor

Leptonic Tensor

\[ L^{\mu\nu} = 2 \left( p^{\mu}_e p^{\nu}_{\nu e} + p^{\nu}_e p^{\mu}_{\nu e} - g^{\mu\nu} p_e \cdot p_{\nu e} - i\epsilon^{\mu\nu\alpha\beta} p_{e\alpha} p_{\nu e\beta} \right) \]

Hadronic Part

\[ W_{\mu\nu} = \frac{1}{2M_B} \sum_{X_c} \langle \bar{B} | J_{q,\nu}^\dagger | X_c \rangle \langle X_c | J_{q,\mu} | \bar{B} \rangle (2\pi)^3 \delta^4(p_B - (p_e + p_{\nu e} + p_{X_c})) \]
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The Basic Idea to Calculate $W_{\mu\nu}$

- Starting point:
  Correlator of two hadronic currents
  
  $$i T_{\mu\nu} = \frac{1}{2M_B} \int d^4x \ e^{-ix(m_b v - q)}$$
  
  $$\times \langle B| \bar{b}_\nu(x) \Gamma^\dagger_\nu c(x) \bar{c}(0) \Gamma_\mu b_\nu(0)|B \rangle$$

- Optical theorem relates $W_{\mu\nu}$ to $T_{\mu\nu}$:
  
  $$-\frac{1}{\pi} \text{Im} \ T_{\mu\nu} = W_{\mu\nu}$$

Momentum Parametrization (HQET)

- Momentum $B$-meson: $P_B = M_B v$

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  $$\Rightarrow$$  
  Momentum of $b$-quark: $p_b = m_b v + k$ with $k \approx O(\Lambda_{QCD})$
Background Field Method

Parametrize Background Field Propagator

- Remove only large momentum: \( p_b = m_b v + k, \ b_v(x) = e^{im_b v \cdot x} b(x) \)
- Background field propagator with \( k \leftrightarrow iD \):

\[
iS_{BGF} = \frac{i}{m_b \gamma - \not{q} + i\not{D} - m_c}
\]

Operator Product Expansion

- HQE corresponds to expand \( S_{BGF} \) in small quantity \( i\not{D} \)

\[
S_{BGF} = \sum_{n=0} \frac{(-1)^n}{\not{Q} - m_c} \left( i\not{D} \frac{1}{\not{Q} - m_c} \right)^n
\]

\[
= \frac{1}{\not{Q} - m_c} - \frac{1}{\not{Q} - m_c} \left( i\not{D} \frac{1}{\not{Q} - m_c} \right) + \frac{1}{\not{Q} - m_c} \left( i\not{D} \frac{1}{\not{Q} - m_c} \right)^2 + \cdots
\]

\( \Rightarrow \) Keeps track on the ordering of the covariant derivatives
Background Field Method

**Parametrize Background Field Propagator**

- Remove only large momentum: \( p_b = m_b v + k, \quad b_v(x) = e^{i m_b v \cdot x} b(x) \)
- Background field propagator with \( k \leftrightarrow iD \):

\[
iS_{\text{BGF}} = \frac{i}{m_b \gamma - \not{q} + i \not{D} - m_c}
\]

**Operator Product Expansion**

- HQE corresponds to expand \( S_{\text{BGF}} \) in small quantity \( i \not{D} \)

\[
S_{\text{BGF}} = \sum_{n=0} (-1)^n \frac{1}{\not{Q} - m_c} \left( i \not{D} \frac{1}{\not{Q} - m_c} \right)^n \\
= \frac{1}{\not{Q} - m_c} - \frac{1}{\not{Q} - m_c} \left( i \not{D} \right) \frac{1}{\not{Q} - m_c} \\
+ \frac{1}{\not{Q} - m_c} \left( i \not{D} \right) \frac{1}{\not{Q} - m_c} \left( i \not{D} \right) \frac{1}{\not{Q} - m_c} + \cdots
\]

\[\Rightarrow\text{ Keeps track on the ordering of the covariant derivatives}\]
The Time-Ordered Product

\[ 2M_B \, T_{\mu\nu} = \langle B(p) | \bar{b}_\nu \gamma_\nu P_L S_{\text{BGF}} \gamma_\mu P_L b_\nu | B(p) \rangle \]

General Structure in each Order

- From the expansion we get a Dirac chain

\[ S_{\text{BGF}}^{(n)} = (Q + m_c) \left[ i\slashed{D} (Q + m_c) \right]^n \frac{1}{(Q^2 - m_c^2 + i\epsilon)^{n+1}} \]

→ "Trace-formulae": Non-perturbative input in Dimension \( n + 3 \)

\[ \langle B(p) | \bar{b}_{\nu,\alpha} (iD_{\mu_1}) \cdots (iD_{\mu_n}) b_{\nu,\beta} | B(p) \rangle = \sum_i \Gamma^{(i)}_{\beta\alpha} A^{(i)}_{\mu_1 \mu_2 \cdots \mu_n} \]

- Start with highest dimension, evaluate recursively
- E.o.m. connect different orders in expansion

→ "Off-shellness" The imaginary part is given by

\[ -\frac{1}{\pi} \, m \, \frac{1}{n!} = \frac{(-1)^n}{n!} (n + 1) \]

\[ \delta^{(n)}(Q^2 - m_c^2) \]
The Time-Ordered Product

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  \]
  \[
  \Rightarrow \text{“Trace-formulae”: Non-perturbative input in Dimension } n + 3
  \]

- \[
  \langle B(p) | \bar{b}_\nu, \alpha (iD_{\mu_1}) \ldots (iD_{\mu_n}) b_\nu, \beta | B(p) \rangle = \sum_i \hat{\Gamma}^{(i)}_{\beta\alpha} A^{(i)}_{\mu_1 \mu_2 \ldots \mu_n}
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  - Start with highest dimension, evaluate recursively
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- “Off-shellness” The imaginary part is given by
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**General Structure in each Order**

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Subtlety in the $1/m_Q$ expansion

- Expansion in both heavy quark masses $m_b$ and $m_c \approx \sqrt{m_b \Lambda}$
- Starting at leading order $\frac{\Lambda^3}{m_b^3} \left( \log \frac{m_c^2}{m_b^2} + \frac{\Lambda^2}{m_c^2} + \ldots \right)$
  - Leading to systematical effects
  - Computation and estimation of higher orders and these effects
- No $1/m_c$ effects for zero-gluon matrix elements
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Non-Perturbative Parameter

To Order $1/m_b^2$

\[
2M_B \hat{\mu}_\pi^2 = -\langle B(p) | \bar{b}_v (iD)^2 b_v | B(p) \rangle \\
\hat{=} \langle \mathbf{p}^2 \rangle
\]

\[
2M_B \hat{\mu}_G^2 = 1/2 \langle B(p) | \bar{b}_v [(iD_\mu), (iD_\nu)] (-i\sigma^{\mu\nu}) b_v | B(p) \rangle \\
\hat{=} \langle \mathbf{s} \cdot \mathbf{B} \rangle
\]

To Order $1/m_b^3$

\[
2M_B \hat{\rho}_D^3 = 1/2 \langle B(p) | \bar{b}_v \left[(iD_\mu), \left[(iv \cdot D), (iD^\mu)\right]\right] b_v | B(p) \rangle \\
\hat{=} \langle \nabla \cdot \mathbf{E} \rangle
\]

\[
2M_B \hat{\rho}_{LS}^3 = 1/2 \langle B(p) | \bar{b}_v \left\{ (iD_\mu), \left[(iv \cdot D), (iD_\nu)\right]\right\} (-i\sigma^{\mu\nu}) b_v | B(p) \rangle \\
\hat{=} \langle \mathbf{s} \cdot \nabla \times \mathbf{B} \rangle
\]
## Non-Perturbative Parameter

### To Order $1/m_b^2$

\[
2M_B \hat{\mu}_2^2 = -\langle B(p) | \bar{b}_\nu (iD)^2 b_\nu | B(p) \rangle \\
\hat{=} \langle p^2 \rangle \\
2M_B \hat{\mu}_G^2 = 1/2 \langle B(p) | \bar{b}_\nu [(iD_\mu), (iD_\nu)] (-i\sigma^{\mu\nu}) b_\nu | B(p) \rangle \\
\hat{=} \langle s \cdot B \rangle
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### To Order $1/m_b^3$

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2M_B \hat{\rho}_D^3 = 1/2 \langle B(p) | \bar{b}_\nu \left[ (iD_\mu), [(iv \cdot D), (iD_\mu)] \right] b_\nu | B(p) \rangle \\
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\]
Higher Orders

- **Dimension - 7: \( 1/m_b^4 \)**
  - 4 Spin independent parameter
  - 5 Spin dependent parameters

- **Dimension - 8: \( 1/m_b^5 \)**
  - **Proliferation of parameters**
  - 8 Spin independent parameter
  - 10 Spin dependent parameter

**Problem in Experiment**
- All parameters have to be extracted from correlated measurements
- Not reliably possible
- Estimate parameters and use this to estimate influence
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Factorization Ansatz

**Ansatz**

- Factorization formulae $|n\rangle$: $B^{(*)}$ states, $Q$ heavy static quark

$$
\langle \bar{B} | \bar{b} \left[ (iD_{\mu_1}) \ldots (iD_{\mu_k}) \right] \left[ (iD_{\mu_{k+1}}) \ldots (iD_{\mu_n}) \right] \frac{1 + \gamma^\nu}{2} \Gamma b | \bar{B} \rangle 
= \sum_n \langle \bar{B} | \bar{b} (iD_{\mu_1}) \ldots (iD_{\mu_k}) Q | n \rangle \cdot \langle n | \bar{Q} (iD_{\mu_{k+1}}) \ldots (iD_{\mu_n}) \Gamma b | \bar{B} \rangle
$$

**Subtlety: Treatment of Timelike Components**

- Time derivative links different orders in $1/m_b$
- Taking the matrix element of the operator

$$i \partial_0 \langle n | \bar{Q} C b(x) | \bar{B} \rangle = -(E_n - M_B) \langle n | \bar{Q} C b(x) | \bar{B} \rangle$$

$$\Rightarrow$$ Time derivative $iv \cdot D$ corresponds to energy difference

$$\bar{\epsilon} = (M_n - m_Q) - (M_B - m_b) \approx 0.4 \text{ GeV}$$

- Finite $m_b$ and assume a generic $\bar{\epsilon}$
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Numerical Approximation

Lowest State Saturation Ansatz

- Decomposition mostly saturated by lowest state
  \[ \langle \bar{B}|\bar{b}_v iD_j iD_k iD_l iD_m \Gamma b_v |\bar{B}\rangle = \langle \bar{B}|\bar{b}_v iD_j iD_k Q|n_0\rangle \langle n_0|\bar{Q} iD_l iD_m \Gamma b_v |\bar{B}\rangle \]

- Time derivative contributes only to
  \[ \langle \bar{B}|\bar{b}_v iD_j(iv \cdot D)^m iD_k \Gamma b_v |\bar{B}\rangle = (-\bar{\epsilon})^m \langle \bar{B}|\bar{b}_v iD_j iD_k Q|n_0\rangle \]

- And in dimension six parameters

Comments

- All higher order terms expressed through
  \[ \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \text{ and } \bar{\epsilon} \]

- Parameters scale as \( \frac{\Lambda^n}{m_b^n} \) as expected ✓
### Numerical Approximation

#### Lowest State Saturation Ansatz

- Decomposition mostly saturated by lowest state
  \[
  \langle \bar{B} | b_v iD_j iD_k iD_l iD_m \Gamma b_v | \bar{B} \rangle = \langle \bar{B} | b_v iD_j iD_k Q | n_0 \rangle \langle n_0 | \bar{Q} iD_l iD_m \Gamma b_v | \bar{B} \rangle
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Measurement Procedure I

Extraction of Heavy Quark Parameters

- Use normalization to cancel out prefactors
- Need completely integrated hadronic phase-space
- Sufficient number of observables for all different parameters

Definition of Observables

- Electron energy spectrum
  \[ \text{BR}(E_e) = \frac{1}{\int \frac{d\Gamma}{dE_e} dE_e} \]

- Moments of electron energy and hadronic invariant mass
  \[ \langle E^n_e M^m_X \rangle (E_{\text{cut}}) = \frac{1}{\int_{E_e > E_{\text{cut}}} \frac{d^2\Gamma}{dE_e dM_X} dE_e dM_X} \int_{E_e > E_{\text{cut}}} E^n_e M^m_X \frac{d^2\Gamma}{dE_e dM_X} dE_e dM_X \]
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2. Moments of electron energy and hadronic invariant mass

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\langle E_e^n M_X^m \rangle (E_{\text{cut}}) = \frac{1}{\int_{E_e > E_{\text{cut}}} \frac{d^2\Gamma}{dE_e dM_X} dE_e dM_X} \int_{E_e > E_{\text{cut}}} E_e^n M_X^m \frac{d^2\Gamma}{dE_e dM_X} dE_e dM_X
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# Measurement Procedure I

## Extraction of Heavy Quark Parameters
- Use normalization to cancel out prefactors
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## Definition of Observables

1. **Electron energy spectrum**

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   \]
Extraction of $V_{cb}$

- Heavy Quark parameters known from fit to moments and spectra
- Normalisation to partial branching fraction determines $|V_{cb}|$

$$
\Gamma = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} f(m_c, m_b, \mu_\pi^2, \ldots)
$$

Remarks

- $E_{\text{cut}}$ restricts phase-space
  - Reduces validity of HQE
- Highly correlated measurement
  - Limits reasonable order of non-perturbative expansion
Extraction of $V_{cb}$

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Measurement Procedure II

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**Remarks**

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Influence of Higher Orders

Reminder on Facts

- Optical theorem $\Rightarrow$ Higher moments sensitive to higher orders
- Experimental measurement fixed
- $\Rightarrow$ Include higher orders will shift heavy quark parameters of lower orders
  - Influence of lower cut on electron energy
  $\Rightarrow$ Up to which scale is the result still valid
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Interested in influence on extracting $|V_{cb}|$
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Generic Effects

Direct effect

- Additional terms in branching ratio
- \( \implies \) Change value of \(|V_{cb}|\) directly

Indirect Effect

- Use estimate of higher-order parameters
- Value fixed by moment \( \mathcal{M}^{(6)} \) up to dimension six
- Compensate effect by change of heavy quark parameter in \( \mathcal{M}^{(6)} \)

\[
\delta m_b = - \frac{\delta \mathcal{M}^{(8)}}{\partial \mathcal{M}^{(6)}}, \quad \delta \mu^2 = - \frac{\delta \mathcal{M}^{(8)}}{\partial \mu^4}, \quad \delta \rho^3_D = - \frac{\delta \mathcal{M}^{(8)}}{\partial \rho^3_D},
\]

\( \implies \) Results in indirect change of \(|V_{cb}|\)

\[
\frac{\delta |V_{cb}|}{|V_{cb}|} = - \frac{1}{2} \frac{1}{\Gamma_{sl}} \frac{\partial \Gamma_{sl}}{\partial \text{HQP}} \delta \text{HQP}
\]
Generic Effects

Direct effect

- Additional terms in branching ratio
  \[ \Rightarrow \text{Change value of } |V_{cb}| \text{ directly} \]

Indirect Effect

- Use estimate of higher-order parameters
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  - Compensate effect by change of heavy quark parameter in \( \mathcal{M}^{(6)} \)
    \[
    \delta m_b = - \frac{\delta \mathcal{M}^{(8)}}{\partial \mathcal{M}^{(6)} / \partial m_b}, \quad \delta \mu_\pi^2 = - \frac{\delta \mathcal{M}^{(8)}}{\partial \mathcal{M}^{(6)} / \partial \mu_\pi^2}, \quad \delta \rho_D^3 = - \frac{\delta \mathcal{M}^{(8)}}{\partial \mathcal{M}^{(6)} / \partial \rho_D^3}
    \]
  \[ \Rightarrow \text{Results in indirect change of } |V_{cb}| \]
  \[
  \frac{\delta |V_{cb}|}{|V_{cb}|} = - \frac{1}{2 \frac{\partial \Gamma_{sl}}{\partial \text{HQP}}} \frac{\partial \Gamma_{sl}}{\partial \text{HQP}} \delta \text{HQP}
  \]
Generic Effects

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⇒ Results in indirect change of $|V_{cb}|$

$$
\frac{\delta |V_{cb}|}{|V_{cb}|} = - \frac{1}{2} \frac{1}{\Gamma_{sl}} \frac{\partial \Gamma_{sl}}{\partial \text{HQP}} \delta \text{HQP}
$$
Generic Effects

Direct effect
- Additional terms in branching ratio
  ⇒ Change value of $|V_{cb}|$ directly

Indirect Effect
- Use estimate of higher-order parameters
- Value fixed by moment $M^{(6)}$ up to dimension six
- Compensate effect by change of heavy quark parameter in $M^{(6)}$

$$\delta m_b = - \frac{\delta M^{(8)}}{\partial M^{(6)} \partial m_b}, \quad \delta \mu^2_\pi = - \frac{\delta M^{(8)}}{\partial M^{(6)} \partial \mu^2_\pi}, \quad \delta \rho^3_D = - \frac{\delta M^{(8)}}{\partial M^{(6)} \partial \rho^3_D}$$

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Direct Effect on Branching Fraction

Naive Assumption

- Definition: $\delta \Gamma_{1/m^k} = \Gamma_{1/m^k} - \Gamma_{1/m^{k-1}}$ and $\Gamma_{\text{parton}}$ leading order

\[
\frac{\delta \Gamma_{1/m^2}}{\Gamma_{\text{parton}}} = -4.3\%
\]
\[
\frac{\delta \Gamma_{1/m^3}}{\Gamma_{\text{parton}}} = -3.0\%
\]
\[
\frac{\delta \Gamma_{1/m^4}}{\Gamma_{\text{parton}}} = 0.75\%
\]
\[
\frac{\delta \Gamma_{1/m^5}}{\Gamma_{\text{parton}}} = 0.6\%
\]
\[
\frac{\delta \Gamma^{IC}}{\Gamma_{\text{parton}}} = 0.7\%
\]

Implication for $|V_{cb}|$

\[
\frac{\delta \Gamma_{1/m^4} + \delta \Gamma_{1/m^5}}{\Gamma_{\text{parton}}} \simeq 1.3\%
\]

$\implies$ Expect direct 0.65% reduction of $|V_{cb}|$
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Indirect Effect on $V_{cb}$ from Selected Moments

Results for $\langle E_e \rangle$

$$\delta m_b = -33 \text{ MeV}, \quad \delta \mu_\pi^2 = -0.39 \text{ GeV}^2, \quad \delta \rho_D^3 = 0.15 \text{ GeV}^3$$

$$\Rightarrow \frac{\delta |V_{cb}|}{|V_{cb}|} = 0.022 \Rightarrow \frac{\delta |V_{cb}|}{|V_{cb}|} = -0.005 \Rightarrow \frac{\delta |V_{cb}|}{|V_{cb}|} = 0.014$$

Results for $\langle M_X^2 \rangle$

$$\delta m_b = -17 \text{ MeV}, \quad \delta \mu_\pi^2 = -0.12 \text{ GeV}^2, \quad \delta \rho_D^3 = 0.086 \text{ GeV}^3$$

$$\Rightarrow \frac{\delta |V_{cb}|}{|V_{cb}|} = 0.011 \Rightarrow \frac{\delta |V_{cb}|}{|V_{cb}|} = -0.0015 \Rightarrow \frac{\delta |V_{cb}|}{|V_{cb}|} = 0.008$$

Combining everything we expect a net increase of $|V_{cb}|$

$$\frac{\delta |V_{cb}|}{|V_{cb}|} \approx + (0.3 \div 0.5)\%$$
### Indirect Effect on $V_{cb}$ from Selected Moments

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Effect of Electron Energy Cut

Legend of different order Contributions

- Blue: $1/m_b^2$
- Green: $1/m_b^3$
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Hadronic Tensor for “IC”

**Starting Point**

\[ W_{\mu\nu} = \frac{1}{2M_B} \sum_{X_c} \langle \bar{B} | J_{q,\nu}^\dagger(x) | X_c \rangle \langle X_c | J_{q,\mu}(0) | \bar{B} \rangle (2\pi)^3 \delta^4(p_B - q - p_{X_c}) \]

**Rewrite for “Intrinsic Charm” Contribution**

- Use translational invariance
  \[ \Rightarrow 2M_B W_{\mu\nu} = \frac{1}{2\pi} \int d^4x \ e^{i(m_b v - q) \cdot x} \langle \bar{B} | J_{q,\nu}^\dagger(x) J_{q,\mu}(0) | \bar{B} \rangle \]

- Expand in local operators
  \[ 2M_B W_{\mu\nu}^{IC} = (2\pi)^3 \delta^4(q - m_b v) \langle \bar{B}(p) | (\bar{b}_v \gamma_{\nu} P_L c) (\bar{c} \gamma_{\mu} P_L b_v) | \bar{B}(p) \rangle \]
  \[ + (2\pi)^3 \left( \frac{\partial}{\partial q_\alpha} \delta^4(q - m_b v) \right) \langle \bar{B}(p) | (i\partial_\alpha \bar{b}_v \gamma_{\nu} P_L c)(\bar{c} \gamma_{\mu} P_L b_v) | \bar{B}(p) \rangle \]
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Naive Differential Rate for “IC” Contribution

Differential Rate

- Decompose IC operator in scalar operators $T_j$

\[
\frac{d^2 \Gamma^{IC}}{dm_X^2 \ dy} = \frac{G_F^2 m_b^5}{24 \pi^3} |V_{cb}|^2 \left( -3 \frac{T_1(m_b)}{m_b^3} \right) \delta(m_X^2) \delta(1 - y)
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Problems

- Double-counting
- Energy conservation: $m_b + 2m_c + \Delta E_{soft} > M_B$

⇒ Need proper definition of effective theory
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## Setup of the Effective Theory

### First Step

- Evolve down from $M_W$ to $m_b$ and integrate out $b$-quark
  - HQET for $b$-quark
    - It remains as a static colour source
    - Product of the two $b \rightarrow c$ currents matches onto a set of local operators at $\mu \approx m_b$
  - Charm-quark still a dynamical degree of freedom

### Second Step for Charm-Quark

- Evolve down from $m_b$ to $m_c$
- Consider different points of view: Integrate out $c$-quark if possible
  - Charm-quark remains as a static source in charm-quark operators
- Calculate proper matching conditions for all operators
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Scenarios

Scenario I

- Consider charm-quark as heavy
  \[ m_b \sim m_c \gg \Lambda_{\text{QCD}} \]

Scenario II

- Consider charm-quark as semi-heavy
  \[ m_b \gg m_c \gg \Lambda_{\text{QCD}} \]

Scenario III

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Scenario I: $m_b \sim m_c \gg \Lambda_{\text{QCD}}$

**Calculable Part**

- Integrate out (hard) quantum fluctuations with virtuality of $\mathcal{O}(m_b, m_c)$

  $\Rightarrow$ Only light-degrees of freedom remain:
  - light quarks
  - gluons
  - quasi-static $b$-quark field in HQET

- Short-distance matching coefficients and phase space integrals are functions of fixed ratio $\rho = m_c^2 / m_b^2$

**Non-Perturbative Part**

- At $\mu < m_c$: Operators with charm-quark do not appear in a standard renormalization scheme like e.g. $\overline{\text{MS}}$

- They correspond to $\langle \bar{B} | \bar{b}_v \ldots c_{\text{static}} \bar{c}_{\text{static}} \ldots b_v | \bar{B} \rangle \equiv 0$

- Matches to zero because of $m_b + 2m_c + \Delta E_{\text{soft}} > m_B$
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Scenario II: \( m_b \gg m_c \gg \Lambda_{\text{QCD}} \)

- First step: Match at high scale \( \mu \sim m_b \)
  - Charm still dynamical and “intrinsic-charm” operators appear in the OPE
- Next step: Use RGE to scale down to semi-hard scale \( \mu_{\text{sh}} \sim m_c \)
  - Integrate out charm-quark and match “intrinsic-charm” operators onto local operators built from light fields as before

2 matching steps

\[
\begin{align*}
\mu &= m_b \\
\mu &= m_c \\
\mu &= \Lambda_{\text{QCD}}
\end{align*}
\]

charm dynamical
charm quasi-static

Difference to Scenario I

- Resum logarithmic terms \( \ln m_c/m_b \) into short-distance coefficient functions
- Expand analytic terms in powers of \( m_c/m_b \sim \sqrt{\Lambda_{\text{QCD}}/m_b} \sim 0.3 \)

\( \Rightarrow \) Reproduces Scenario I
Scenario II: $m_b \gg m_c \gg \Lambda_{QCD}$

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Mixing of Operators

**Dimension 6 Intrinsic Charm**

- Generates mixing into $\rho_D^3$
- $\Rightarrow$ Renormalization group flow

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\frac{d}{d \ln \mu} \begin{pmatrix} \rho_D(\mu) \\ T_1(\mu) \\ T_2(\mu) \end{pmatrix} = - \begin{pmatrix} 0 & 0 & 0 \\ -2/3 & 0 & 0 \\ 4/3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \rho_D(\mu) \\ T_1(\mu) \\ T_2(\mu) \end{pmatrix}
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**Dimension 7 Intrinsic Charm**

- Generates mixing into $m_c^4 \bar{b}_v b_v$
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$1/m_c^{2n}$ terms also reproduced [hep-ph/0511158]
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Scenario III: $m_b \gg m_c \gtrsim \Lambda_{QCD}$

- Charm-quark effects cannot be integrated out perturbatively
  $\Rightarrow$ Define proper power counting

Consequences

- Genuine intrinsic-charm operators exist
  $\Rightarrow$ Hadronic matrix elements of this operators have to be defined at $\mu_0$ with $m_b \geq \mu_0 \gg m_c$

- Matrix elements contain non-analytic dependence on $m_c$
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Model: Weak Annihilation in $b \rightarrow u$ Transitions

- Blue: Leading Log from order $1/m_b^3$
- Yellow: Including $1/(m_b^3 m_c^2)$ Corrections
- Red: Model (s.b.)

Model for “Weak-Annihilation” Operator

- Szenario 3: Four quark operator appears: “Weak-Annihilation“
- Renormalization group inspired model

$$\frac{1}{2M_B} \langle B| \bar{b}\gamma^k(1 - \gamma_5)c \bar{c}\gamma^k(1 - \gamma_5)b|B\rangle = \frac{\rho_D^3}{m_b^3} \ln \frac{m_c^2}{m_b^2 + \Lambda^2}$$

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  - Using factorization schema for non-perturbative parameters
  - Very good convergence of HQE
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  - Two scenarios with heavy charm quark $\Rightarrow$ Equal results
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- Non-valenz contribution $\mathcal{O}(3\%)$ in total rate of $B \rightarrow X_u \ell \bar{\nu}_\ell$
Summary

- Computed corrections of expansion up to $O(1/m_b^5)$ analytically
- Numerical analysis presented
  - Using factorization schema for non-perturbative parameters
  - Very good convergence of HQE
- Error in $|V_{cb}|$ from non-perturbative corrections $O(1\%)$
  ⇒ Cannot explain „tension“ exclusive vs inclusives $|V_{cb}|$
- Investigation of systematics with charm quark
  - Two scenarios with heavy charm quark ⇒ Equal results
  - Light final state quark: Additional operator
- Model estimate of four quark operator
  - One model parameter: Fitted by comparison with $1/m_b^5$ expansion
  - Non-valenz contribution $O(3\%)$ in total rate of $B \to X_u \ell \bar{\nu}_\ell$