Conformal symmetry constraints on the renormalisation of heavy-light light ray operators

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Light-Ray Operators

- light-ray operators (simple example)

$$\mathcal{O}(z_1, z_2) = \bar{q}(z_1)[z_1, z_2]\Gamma q(z_2)$$

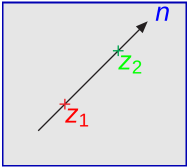
- heavy-light operators

$$\mathcal{O}_h(z_1, z_2) = h_v^*(z_1)[z_1, z_2]\Gamma q(z_2)$$

q: light quark fields

$z_i^\mu = z_i n^\mu$

light-like vectors n, $\tilde{n}$



- more general

$$\mathcal{O}_h(z_1, \ldots, z_N) = S(\Phi(z_1)\ldots\Phi(z_N)), \quad \Phi(z_i) = [0, z_i]\Phi(z_i)$$

S: color tensor, Φ: heavy or light field, for heavy quark $z_i \to 0$

Phenomenological Applications

- factorization theorems in exclusive decays

$$
\begin{aligned}
B \to \pi l \nu \quad &\longrightarrow \quad F_i = C_i \xi_\pi + \phi_B \star \mathcal{T}_i \star \phi_\pi \\
B \to \pi\pi \quad &\longrightarrow \quad F_i \mathcal{T}_i^1 \star \phi_\pi + \phi_B \star \mathcal{T}_i^2 \star \phi_\pi \star \phi_\pi \\
\Lambda_b \to \Lambda\gamma \quad &\longrightarrow \quad \Psi_\Lambda \star \mathcal{T} \star \Psi_{\Lambda_b}
\end{aligned}
$$

F_i: form factors $\mathcal{T}_i$ scattering amplitudes

- distribution amplitudes

$$
\begin{aligned}
\langle 0|\bar{q}(z_1)[z_1, z_2]\Gamma q(z_2)|\pi(p)\rangle &\sim \phi_\pi^{(2)},\ \phi_{\pi,p},\ \phi_{\pi,\sigma} \ldots \\
\langle 0|h_v^*(0)[0, z]\Gamma q(z)|B(p)\rangle &\sim \phi_B^+(z),\ \phi_B^-(z) \\
\epsilon^{abc}\langle 0|q^a(z_1)C\Gamma q^b(z_2)h_v^c(0)|\Lambda_b(p)\rangle &\sim \Psi_2,\ \Psi_3^s,\ \Psi_3^\sigma,\ \Psi_4
\end{aligned}
$$

Distribution amplitudes classified by twist of light degrees of freedom

- mainly $\phi_\pi^{(2)}$, ϕ_B^+ and Ψ_2 needed

Conformal Invariance

- conformal group is extension of Poincaré-group that leaves the light-cone invariant $x^2 = 0$
- includes Lorentz-transformations, translations, scale transformations D, special conformal transformations K^μ [15 generators]

$$x^\mu \longrightarrow x'^\mu = \lambda x^\mu, \qquad x^\mu \longrightarrow x'^\mu = \frac{x^\mu + a^\mu x^2}{1 + 2a \cdot x + a^2 x^2}$$

- action S of massless QCD is conformally invariant
- conformal symmetry is broken at one-loop level due to conformal anomaly

$$\delta_D S \sim (D-4) \int d^4x \left[\frac{1}{4} G_{\mu\nu} G^{\mu\nu}(x) + \cdots \right]$$

$$\delta_K^\alpha S \sim (D-4) \int d^4x 2x^\alpha \left[\frac{1}{4} G_{\mu\nu} G^{\mu\nu}(x) + \cdots \right]$$

- one-loop counterterms inherit symmetry from the action

Collinear Conformal Group SL(2,$\mathbb{R}$)

- for fields on the light-cone $z_i = z_i n$, $n^2 = 0$ conformal group reduces to collinear conformal group SL(2,$\mathbb{R}$)
- SL(2,$\mathbb{R}$) has three generators : translation S_-, special conformal transformation S_+ and dilatation plus Lorentz-rotation S_0

$$[S_0, S_\pm] = \pm S_\pm, \qquad [S_+, S_-] = -2S_0$$

- fields (and operators) can be classified by conformal spin (dimension plus spin projection on the light-cone) and conformal twist (dimension minus spin projection on the light-cone)
- renormalization of light-light light-ray operators which furnish representation of SL(2,$\mathbb{R}$) has to commute with group transformations
- useful for classification: spinor representation of fields

Spinor Notation I

- coordinates

$$x_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} x_{\mu} = \begin{pmatrix} x_0 + x_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_0 - x_3 \end{pmatrix} = \begin{pmatrix} x_+ & w \\ \bar{w} & x_- \end{pmatrix}, \qquad \sigma^{\mu} = (1, \vec{\sigma})$$

- introduce two light-like vectors $n^2 = \tilde{n}^2 = 0$ with auxiliary spinors λ, μ

$$n_{\alpha\dot{\alpha}} = \lambda_{\alpha}\bar{\lambda}_{\dot{\alpha}}, \qquad \tilde{n}_{\alpha\dot{\alpha}} = \mu_{\alpha}\mu_{\dot{\alpha}}$$

so that

$$x_{\alpha\dot{\alpha}} = z\lambda_{\alpha}\bar{\lambda}_{\dot{\alpha}} + \tilde{z}\mu_{\alpha}\bar{\mu}_{\dot{\alpha}} + w\lambda_{\alpha}\bar{\mu}_{\dot{\alpha}} + \bar{w}\mu_{\alpha}\bar{\lambda}_{\dot{\alpha}}$$

Spinor Notation II

- fields

$$q = \begin{pmatrix} \psi_\alpha \\ \bar\chi^{\dot\beta} \end{pmatrix}, \qquad \bar q = (\chi^\beta, \bar\psi_{\dot\alpha})$$

$$F_{\alpha\beta,\dot\alpha\dot\beta} = \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta}F_{\mu\nu} = 2\left(\epsilon_{\dot\alpha\dot\beta}f_{\alpha\beta} - \epsilon_{\alpha\beta}\bar f_{\dot\alpha\dot\beta}\right)$$

- plus/minus-components

$$\begin{array}{lll} \psi_+ = \lambda^\alpha\psi_\alpha & \chi_+ = \lambda^\alpha\chi_\alpha & f_{++} = \lambda^\alpha\lambda^\beta f_{\alpha\beta} \\ \bar\psi_+ = \bar\lambda^{\dot\alpha}\bar\psi_{\dot\alpha} & \bar\chi_+ = \bar\lambda^{\dot\alpha}\bar\chi_{\dot\alpha} & \bar f_{++} = \bar\lambda^{\dot\alpha}\bar\lambda^{\dot\beta}\bar f_{\dot\alpha\dot\beta} \\ \psi_- = \mu^\alpha\psi_\alpha & \bar\psi_- = \bar\mu^{\dot\alpha}\bar\psi_{\dot\alpha} & f_{+-} = \lambda^\alpha\mu^\beta f_{\alpha\beta} \end{array}$$

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- plus/minus-components

	ψ_+	f_{++}	ψ_-	f_{+-}	$\bar{D}_{-+}\psi_+$	$\bar{D}_{-+}f_{++}$
j	1	3/2	1/2	1	3/2	2
E	1	1	2	2	2	2
H	1/2	1	−1/2	0	3/2	2

j,E,H: conformal spin, Twist, helicity

same quantum numbers for antichiral fields except helicity

- operators build out of light fields furnish representation of $SL(2,\mathbb{R})$ with definite conformal spin and twist

General Considerations

- a generic light-ray operator mixes under renormalization with operators with the same quantum numbers

$$\mathcal{O}(\Phi)_i = Z_{ik}\mathcal{O}_k(\Phi)$$

- anomalous dimension

$$\gamma = -\mu \frac{d}{d\mu}\mathbb{Z}\mathbb{Z}^{-1}, \qquad \gamma = \frac{\alpha_s}{2\pi}\mathcal{H}$$

is of block diagonal form

operators with N fields can go to operators with N+1 fields not other way round

- diagonal elements consist at one-loop out of products of $2 \to 2$-kernels
- off-diagonal elements consist at one-loop out of $2 \to 3$-kernels

light-light case

- operators of different twist do not mix
- $\mathcal{H}$ constrained by conformal symmetry

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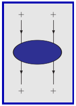
heavy-light case

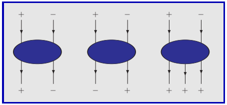
- ?
- ?

Light-Light/Heavy-Light Case

- quasi-partonic operators
  - operators build of “plus”-components Twist $=$ number of fields
  - closed under renormalisation
  - kernels known for long time



- non-quasi-partonic operators
  - mix under renormalisation with quasi-partonic operators
  - renormalisation can be reconstructed from quasipartonic case by conformal symmetry



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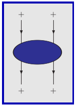

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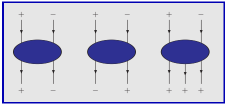
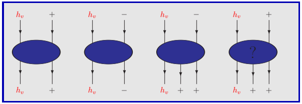


- goal of this project
  - mixing and symmetry constraints for heavy-light operators
  - which symmetry holds?



Symmetries For The Heavy-Light Case

- heavy quark does not furnish any representation of the conformal group but can be represented as Wilson-line along v^μ-direction times a sterile field

$$h_v(0) = P \exp\left\{ i g_s \int_{-\infty}^{0} d\alpha v^\mu A_\mu(\alpha v) \right\} \phi(-\infty)$$

$$v^2 = 1$$

- line is left invariant by dilatation D
- line is left invariant by special conformal transformation along v^μ-direction

$$x^\mu = x v^\mu : \quad x'^\mu = \frac{x v^\mu + x^2 v^\mu}{1 + 2x + x^2} = \frac{x}{1+x} v^\mu$$

- for fields living on the light-cone $v \cdot K$ reduces to S_+
- problem, with static heavy quark

$$[v \cdot K, \mathcal{H}]\mathcal{O}_h(z) = 0, \qquad [D, \mathcal{H}]\mathcal{O}_h(z) = 0$$

only possible for a constant

$\mathcal{O}_h = h_v(0)\Phi(z)$

- contradicts known results

Interpretation I

- take path integral

$$\int [D\Phi] \big(\mathcal{O}(\Phi) + \Delta\mathcal{O}(\Phi) \big) e^{-S_R[\Phi] + i \int d^4x \, \Phi \, \lambda} = \textit{finite}$$

$\Delta\mathcal{O}(\Phi)$: counterterm
λ: sources

- change of variables $\Phi \to \Phi + \delta_\alpha \Phi$ does not change integral

$$\int [D\Phi] \exp\left\{ iS_R + i \int d^4x \, \Phi \, \lambda \right\} (\delta_\alpha \mathcal{O}(\Phi) + \delta_\alpha \Delta\mathcal{O}(\Phi))$$
$$= \int [D\Phi] \exp\left\{ iS_R + i \int d^4x \, \Phi \, \lambda \right\} \left(\delta_\alpha S_R - i \int d^4x \lambda \delta_\alpha \Phi \right)$$
$$\times \, (\mathcal{O}(\Phi) + \Delta\mathcal{O}(\Phi))$$

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$$\int [D\Phi]\big(\mathcal{O}(\Phi) + \Delta\mathcal{O}(\Phi)\big) e^{-S_R[\Phi]+i\int d^4x\,\Phi\,\lambda} = finite$$

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$$= \int [D\Phi] \exp\left\{iS_R + i\int d^4x\,\Phi\,\lambda\right\}\left(\delta_\alpha S_R - i\int d^4x\lambda\delta_\alpha\Phi\right)$$
$$\times\,(\mathcal{O}(\Phi) + \Delta\mathcal{O}(\Phi))$$

- red terms on right hand side determine difference $\delta_\alpha\Delta\mathcal{O} - \Delta\delta_\alpha\mathcal{O}$

$$\delta_\alpha\Delta\mathcal{O} - \Delta\delta_\alpha\mathcal{O} = \int [D\Phi] \exp\left\{iS_R + i\int d^4x\,\Phi\,\lambda\right\}\delta_\alpha S_R \times \mathcal{O}(\Phi)$$

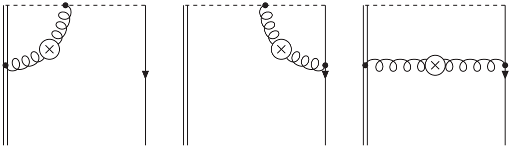
A Short Illustration

- calculation of

$$\delta_\alpha \Delta\mathcal{O} - \Delta\delta_\alpha\mathcal{O} = \int [D\Phi] \exp\left\{ iS_R + i\int d^4x\, \Phi\, \lambda \right\} \delta_\alpha S_R \times \mathcal{O}(\Phi)$$

$\mathcal{O}(\Phi) = h_v\, \psi_+(z)$ and $\delta_\alpha = \alpha\, \delta_D$

in Feynman-gauge to $\mathcal{O}(\alpha_s)$ amounts to calculating the following diagrams



$\otimes$: $(D-4)\int d^D x\, G_{\mu\nu}\, G^{\mu\nu}(x)$

A Short Illustration

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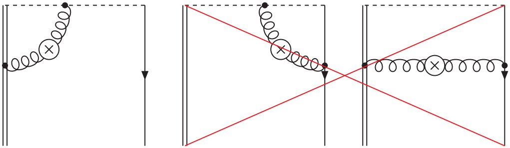


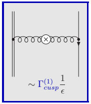
$\otimes$: $(D-4)\int d^Dx\, G_{\mu\nu}\, G^{\mu\nu}(x)$

- two diagrams do not contribute for gauge invariant operators

omit subtleties related to variation of gauge-fixing terms

- remaining diagram $\sim \Gamma^{(1)}_{cusp}\,\frac{1}{\epsilon}$

calculation in light-cone gauge more appropriate for our purpose but less instructive



Interpretation II

- therefore counterterm $\Delta\delta_D\mathcal{O}(\Phi)$

$$\Delta\delta_D\mathcal{O}(\Phi) = \delta_D\Delta\mathcal{O}(\Phi) + \Gamma^{(1)}_{Cusp}\frac{1}{\epsilon}\mathcal{O}(\Phi)$$

$\Gamma^{(1)}_{Cusp}$ responsible for breaking of scale invariance in one-loop renormalization of heavy-light operators

- in other language symmetry relations for heavy-light renormalization kernels:

$$\begin{aligned} [D,\mathcal{H}]\mathcal{O}_h(z) &= \Gamma^{(1)}_{Cusp}\mathcal{O}_h(z) \\ [v\cdot K,\mathcal{H}]\mathcal{O}_h(z) &= 0 \end{aligned}$$

relations for gauge-variant operators only valid in light-cone gauge

- one-loop $2 \to 2$-renormalization fixed up to a constant by symmetry

General Form

- action of symmetry generators can be represented by differential operators

$$v \cdot K\,\mathcal{O}(z) = (z^2\partial_z + 2jz)\mathcal{O}(z), \quad D\mathcal{O}(z) = (z\partial_z + l)\mathcal{O}(z)$$

j : conformal spin of light field

l : canonical dimension of light field

- $[D, \mathcal{H}]\mathcal{O}_h(z) = \Gamma^{(1)}_{Cusp}\mathcal{O}_h(z)$:

$$\Rightarrow [\mathcal{H}\,\mathcal{O}_h](z) = C\left(\int_0^1 d\alpha f(\bar{\alpha})\mathcal{O}_h(\bar{\alpha}z) + \Gamma^{(1)}_{Cusp}\log(i\mu z)\mathcal{O}_h(z) + A\mathcal{O}_h(z)\right)$$

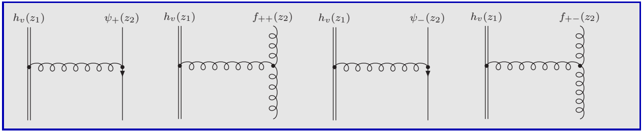
C: color-structure

- $[v \cdot K, \mathcal{H}]\mathcal{O}_h(z) = 0$:

$$\Rightarrow f(\bar{\alpha}) = \left(\frac{-\bar{\alpha}^{2j-1}}{\alpha}\right)_+$$

Explicit Calculation

- calculating in coordinate space
- using light-cone gauge
- $2 \to 2$-kernels



- confirms theoretical considerations

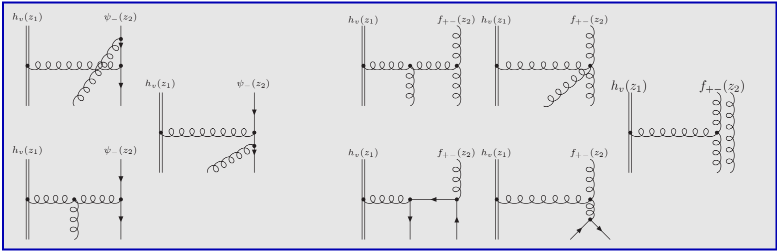
$$[\mathcal{H}\,\mathcal{O}_h](z) = C\left[\int_0^1 \frac{d\alpha}{\alpha}\left(\mathcal{O}_h(z) - \bar{\alpha}^{2j-1}\mathcal{O}_h(\bar{\alpha}z)\right) + \Gamma^{(1)}_{cusp}\log(i\mu z)\mathcal{O}_h(z) - \sigma_h\mathcal{O}_h(z) - \sigma_{q(g)}\mathcal{O}_h(z)\right]$$

$\sigma_{q(g)}$: quark- or gluon-field renormalization constant

σ_h: heavy-quark renormalization constant

Explicit Calculation II

- calculating again in coordinate space
- and using light-cone gauge
- $2 \longrightarrow 3$-kernels for $h_v \psi_-$ and $h_v f_{+-}$



- results coincide with the light-light case if h_v is substituted by ψ_+
- Γ_{cusp} appears only in $2 \longrightarrow 2$ kernels
- operator-mixing purely governed by light-degrees of freedom

Conclusions

- $2 \to 2$-kernels fixed up to a constant by conformal symmetry
  - used properties of Wilson-line in v-direction under conformal transformations
  - exhibit general form for all light degrees of freedom
- $2 \to 3$-kernels coincide with light-light case if h_v is substituted by ψ_+
  - no $\frac{1}{\epsilon^2}$ or $\frac{1}{\epsilon}\log(i\mu z)$ terms
  - mixing seems to be governed solely by light degrees of freedom
  - no extraordinary mixing seen
- $2 \to 2$- and $2 \to 3$-kernels can be used to build renormalization of generic many particle operators including a heavy quark

B-meson/Λ_b Distribution Amplitudes

- B-meson distribution amplitudes
- $h_v^*(0)\not{n}\gamma_5 q(z) \longrightarrow h_v(0)\psi_+(z)$ $\quad\phi_+^B$
- $h_v^*(0)\not{\tilde{n}}\gamma_5 q(z) \longrightarrow h_v(0)\psi_-(z)$ $\quad\phi_-^B$
- $h_v^*(0)G_{\mu\nu}n^\nu\gamma_\perp^\mu\not{n}\gamma_5 q(z) \longrightarrow h_v(0)\bar{f}_{++}(uz)\chi_+(z)$ $\quad\Psi_A - \Psi_V$
- $h_v^*(0)G_{\mu\nu}n^\nu\gamma_\perp^\mu\not{\tilde{n}}\gamma_5 q(z) \longrightarrow h_v(0)f_{++}(uz)\chi_-(z)$ $\quad\Psi_A + \Psi_V$

- Λ_b-distribution amplitudes
- $\epsilon^{ijk}u^{T,i}(z_1)C\gamma_5\not{n}d^j(z_2)h_v^k(0) \longrightarrow \epsilon^{ijk}\psi_+^i(z_1)\bar{\chi}_+^j(z_2)h_v^k(0)$ $\quad\Psi_2$
- $\epsilon^{ijk}u^{T,i}(z_1)C\gamma_5 d^j(z_2)h_v^k(0) \longrightarrow \epsilon^{ijk}\psi_+^i(z_1)\psi_-^j(z_2)h_v^k(0)$ $\quad\Psi_3^p$
- $\epsilon^{ijk}u^{T,i}(z_1)C\gamma_5\sigma_{\mu\nu}n^\mu\tilde{n}^\nu d^j(z_2)h_v^k(0) \longrightarrow \epsilon^{ijk}\psi_-^i(z_1)\psi_+^j(z_2)h_v^k(0)$ $\quad\Psi_3^\sigma$