Double Ratios in (Rare) Semi-Leptonic Decays and New Physics

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in collaboration with
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Preliminary Work

Lawrence Berkeley National Laboratory

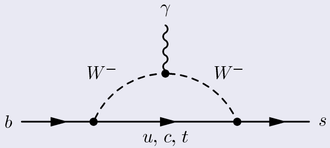
Colour meets Flavour
Saturday, October 14th, 2011

Outline

Rare Decays and New Physics

Rare Decays

- SM very successful
- Deviations mostly expected in flavour changing neutral currents



Influence of New Physics

- Integrate out heavy degress of freedom
- Encoded in Wilson coefficients of operator product expansion

⇒ New physics manifests in Wilson coefficients, only.
Assumption: Complete basis in Standard Model (SM)

- Search for deviations of predicted SM Wilson coefficients

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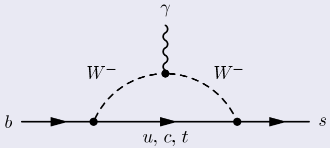

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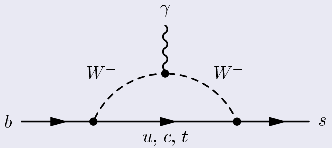

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The Goal

Current New Physics Searches: One example

- Investigate $b \to s$ transition
  1. $b \to s\gamma$
  2. $b \to s\ell^+\ell^-$
- Constrain possible New Physics (NP) contribution by [arXiv:1106.1547]
  1. Forward-Backward Asymmetry $\mathcal{A}_{\mathrm{FB}}(q^2)$
  2. Angular analysis $B \to K^*(K\pi)\ell^+\ell^-$ for CP violation

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- Consider $B \to K^*\ell^+\ell^-$
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Wilson Coefficients and Theoretical Uncertainties

Wilson Coefficients

- $\mathcal{O}_1 - \mathcal{O}_8$:Contribute indirectly due to long-distance quark loops

⇒ Lots of investigations [JHEP 1009, 089 (2010),Eur. Phys. J. C 71, 1635 (2011)]

- $\mathcal{O}_7$: Wilson coefficient best constraint by $b \to s\gamma$ at low q^2
- $\mathcal{O}_9 - \mathcal{O}_{10}$: Semi-leptonic operators

⇒ C_9 and C_{10} sensitive to new physics in $b \to s\ell^+\ell^-$ at high q^2 different to $\mathcal{O}_7$

Non-perturbative Input

- Observing bound state mesons

$$\langle K^*(p_{K^*})|\mathcal{O}_i^{\mu_1\ldots\mu_n}|B(p_B)\rangle = f(p_{K^*}^{\mu_j}, p_B^{\mu_j}, g^{\mu_j\mu_{j'}})$$

⇒ Decomposition in scalar form factors

⇒ Largest theoretical uncertainties

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Double Ratios in a Simple Example

- Consider ratios of decay constants [PRL 71, 3067, PRD 53, 4937]

Parametric deviations from unity

1. Chiral $SU(3)$ light flavor symmetry
2. Heavy-quark spin-flavor symmetry $SU(4)$
3. Perturbative corrections

- Compute the double ratio $[f_{B_s}/f_B]/[f_{D_s}/f_D]$

⇒ Lots of theoretical uncertainties cancel

$$\frac{f_{B_s}/f_B}{f_{D_s}/f_D} \approx 1 + \mathcal{O}\left(\frac{m_s}{m_c} - \frac{m_s}{m_b}, \frac{m_s}{\Lambda}\frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right) \approx 1 + \mathcal{O}(\lesssim 7\%)$$

Depending on the point of view

1. Precise extraction of one CKM matrix element
2. Extract one of the four decay constants to a higher precision

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The Double Ratio Proposal

Suggestion

- Use the ratio of $(0 \leq q^2 \leq (M_{\text{initial}} - M_{\text{final}})^2)$

$$\frac{\int_{q_0^2}^{q_{\max}^2} \frac{d\Gamma}{dq^2}(B \to K^* \ell^+ \ell^-)}{\int_{q_0^2}^{q_{\max}^2} \frac{d\Gamma}{dq^2}(B \to \rho \ell \bar{\nu}_\ell)} \frac{\int_{q_0^2}^{q_{\max}^2} \frac{d\Gamma}{dq^2}(D \to \rho \ell \bar{\nu}_\ell)}{\int_{q_0^2}^{q_{\max}^2} \frac{d\Gamma}{dq^2}(D \to K^* \ell \bar{\nu}_\ell)}$$

$$= \#(C_i) + \mathcal{O}\left(\frac{m_s}{m_c} - \frac{m_s}{m_b}, \frac{m_s}{\Lambda_{\text{QCD}}} \frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right)$$

- All decays are pseudoscalar to vector transitions
- $B \to K^*$: FCNC, others are tree-level

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Problems

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Computational Methods for Form Factors

Brief Comment

- Parameterize q^2 dependence, usually pole form

$$F(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_{\text{fit}}^2},$$

  - 1 Vector $V(q^2)$
  - 3 Axialvector $A_{0-2}(q^2)$, Most important: $A_1(q^2)$ at high q^2
  - 3 Tensor $T_{1-3}(q^2)$ form factors (only rare decay)
- Compute specific values with various methods, e.g. LCSR, lattice

$\Rightarrow$ Fit to parameterization

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Theoretical Properties

1. Heavy quark symmetry predicts scaling behaviour and relations between form factors of a specific decay
Form factors differently defined in HQE application!

$$A_1 \propto \mathcal{O}(m_b^{-1/2} + m_b^{-3/2})$$

2. Heavy quark symmetry: Corrections between B and D decays
3. Chiral symmetry: Corrections between B/B_s and D/D_s decays

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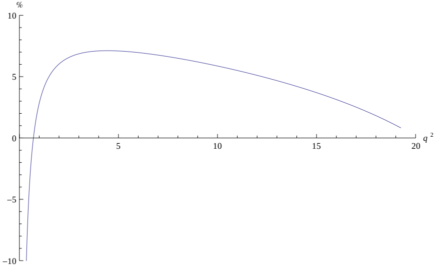
- $\frac{h - \frac{a_+ - a_-}{2m_b} - \frac{g}{m_b}}{\frac{g}{m_b}}$
- $\frac{\frac{f}{m_b} + \left(M_B^2 + M_{K*}^2 - q^2\right)\frac{g}{m_b} - (g_- + g_+)}{\left(M_B^2 + M_{K*}^2 - q^2\right)\frac{g}{m_b}}$
- $\frac{2gm_b - (g_- - g_+)}{g_+ - g_-}$
- Expected from HQE $10 - 15\%$



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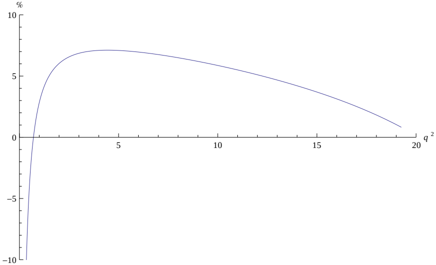

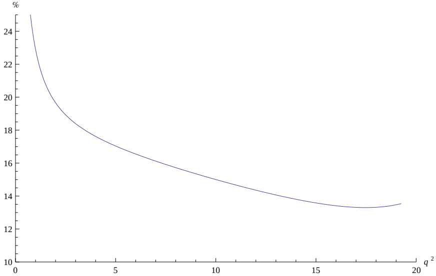

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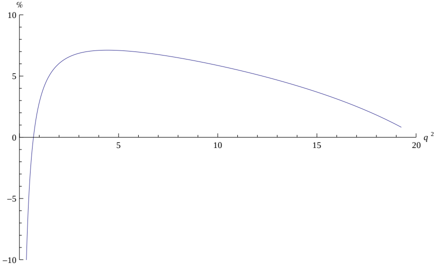

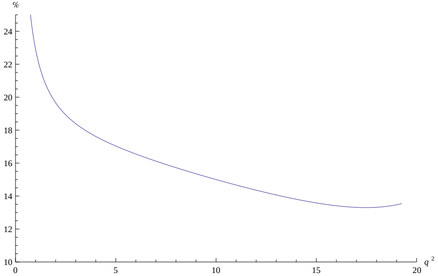

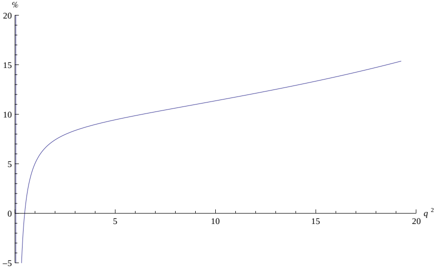

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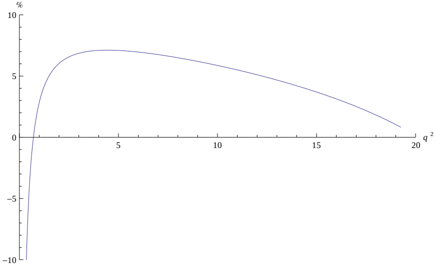
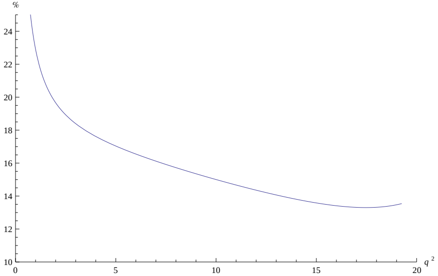
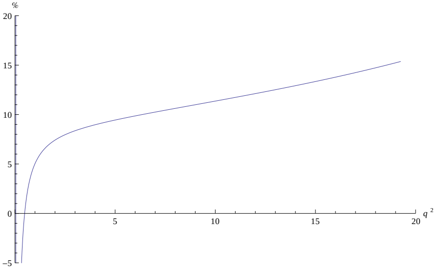
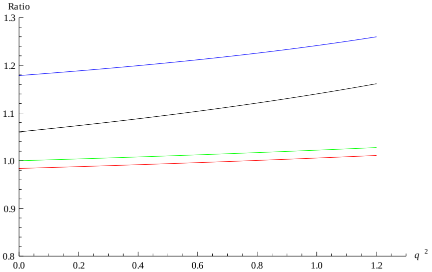






Ratio of Meson Decay Form Factors

$$\frac{FF^{D\to\rho}(q^2)}{FF^{D\to K^*}(q^2)}$$



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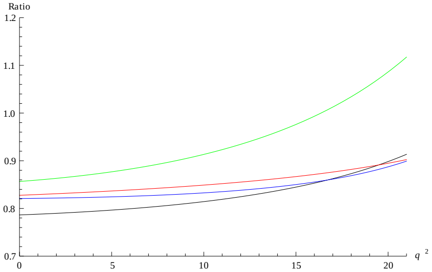

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Double Ratio of Form Factors

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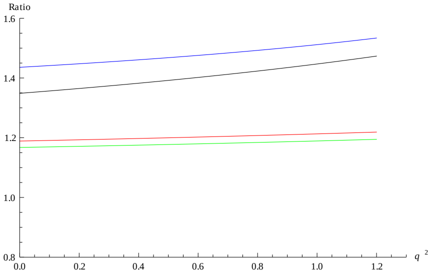

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Remarks

- Ratio for D decay range of q^2!
- Improves for choosing different q^2 integration ranges
- Most important $A_1(q^2) \Rightarrow \mathcal{O}(10-20\%)$ possible

Numerical Result for Double Ratio

- Use natural HQE variable $w = \frac{1}{2M_i M_j}\left(M_i^2 + M_j^2 - q^2\right)$
- B decays: $q^2 \gtrsim 15\,\text{GeV}^2 \Rightarrow 1 \leq w \leq w_B = 1.45$
- D decays: Full $D \to K^*$ phase-space $\Rightarrow 1 \leq w \leq w_B = 1.29$

$$
\begin{aligned}
\text{DR}(w_D, w_B) = \frac{3\alpha^2}{64\pi^2}\frac{|V_{cd}|^2|V_{tb}|^2|V_{ts}|^2}{|V_{cs}|^2\,|V_{ub}|^2}\Big[&86.55|C_1|^2 + 9.62|C_2|^2 + 53.2|C_3|^2 + 61.96|C_4|^2 \\
&+ 66.14|C_5|^2 + 7.35|C_6|^2 + 14.55|C_7|^2 + 4.51|C_9|^2 + 4.51|C_{10}|^2 \\
&+ 57.7\,\text{Re}\,(C_1\,C_2{}^*) + 77.48\,\text{Re}\,(C_1\,C_3{}^*) - 100.6\,\text{Re}\,(C_1\,C_4{}^*) \\
&+ 124.9\,\text{Re}\,(C_1\,C_5{}^*) + 41.64\,\text{Re}\,(C_1\,C_6{}^*) + 38.94\,\text{Re}\,(C_1\,C_7{}^*) \\
&+ 16.18\,\text{Re}\,(C_1\,C_9{}^*) + 25.82\,\text{Re}\,(C_2\,C_3{}^*) - 33.54\,\text{Re}\,(C_2\,C_4{}^*) \\
&+ \ldots \\
&+ 21.6\,\text{Re}\,(C_7\,C_9{}^*) - 111.34\,\text{Im}\,(C_1\,C_3{}^*) - 106.38\,\text{Im}\,(C_1\,C_4{}^*) \\
&- 85.36\,\text{Im}\,(C_1\,C_5{}^*) - 28.46\,\text{Im}\,(C_1\,C_6{}^*) - 86.3\,\text{Im}\,(C_1\,C_7{}^*) \\
&- 36.\,\text{Im}\,(C_1\,C_9{}^*) - 37.12\,\text{Im}\,(C_2\,C_3{}^*) - 35.46\,\text{Im}\,(C_2\,C_4{}^*) \\
&+ \ldots
\end{aligned}
$$

Summary

- Showed potential of double ratios
- Application to rare semi-leptonic decay at high q^2
- Showed first numerical estimate of uncertainties
- Presented numerical result of double ratio as function of Wilson coefficients
- Analysis with uncertainties $\mathcal{O}(10-20\%)$ possible

Outlook

- Wait for new experimental data on D decay form factors
- Perform more elaborate numerical analysis
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Backup Slides

A Little Bit Notation: Operator Basis

- Consider rare decay: $B \to K^* \ell^+ \ell^-$
- $\mathcal{O}_1 - \mathcal{O}_2$: Current-current operators
- $\mathcal{O}_3 - \mathcal{O}_6$: QCD-penguin operators
- $\mathcal{O}_7 - \mathcal{O}_8$: Magnetic penguin operators
- $\mathcal{O}_9 - \mathcal{O}_{10}$: Semi-leptonic operators

$$\mathcal{O}_1^p = (\bar{p}b)_{V-A}(\bar{s}p)_{V-A} \qquad \mathcal{O}_2^p = (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A}\,,$$

$$\mathcal{O}_3 = (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V-A} \qquad \mathcal{O}_4 = (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V-A}\,,$$

$$\mathcal{O}_5 = (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V+A} \qquad \mathcal{O}_6 = (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V+A}\,,$$

$$\mathcal{O}_7 = \frac{e}{8\pi^2}\, m_b\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}\, b \qquad \mathcal{O}_8 = \frac{g}{8\pi^2}\, m_b\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}\, b\,,$$

$$\mathcal{O}_9 = \frac{\alpha}{2\pi}(\bar{s}b)_{V-A}(\bar{l}l)_V \qquad \mathcal{O}_{10} = \frac{\alpha}{2\pi}(\bar{s}b)_{V-A}(\bar{l}l)_A\,.$$

Comment on Operators and Wilson Coefficients

C_1 to C_8

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- Contribute indirectly due to long-distance quark loops
- Problems: Resonances, Duality violation, ...

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⇒ C_9 and C_{10} most sensitive to new physics in $b \to s\ell^+\ell^-$

Comment on Operators and Wilson Coefficients

C_1 to C_8

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- $\mathcal{O}_3 - \mathcal{O}_6$: QCD-penguin operators
- $\mathcal{O}_7 - \mathcal{O}_8$: Magnetic penguin operator
- Contribute indirectly due to long-distance quark loops
- Problems: Resonances, Duality violation, ...

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The Tree Level Decays

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2|V_{ji}|^2}{192\pi^3 M_i^3}\sqrt{(M_i^2+M_j^2-q^2)^2-4M_i^2M_j^2}\left((M_i+M_j)^2-q^2\right)$$
$$\left(q^2\left((M_i+M_j)^2+M_i^2+M_j^2-4q^2\right)+\left(M_i^2-M_j^2\right)^2\right)[F_{ij}(q^2)]^2$$

with

$$[F_{ij}(q^2)]^2 = \frac{M_i((M_i-M_j)^2-q^2)}{M_j(M_i+M_j)^2\left(q^2\left(3M_i^2+2M_iM_j+3M_j^2\right)+\left(M_i^2-M_j^2\right)^2-4q^{2\,2}\right)}$$
$$\Bigg[8M_j^2q^2\left([V^{ij}(q^2)]^2+\frac{(M_i+M_j)^4}{M_i^4-2M_i^2\left(M_j^2+q^2\right)+\left(M_j^2-q^2\right)^2}[A_1^{ij}(q^2)]^2\right)$$
$$+\left(M_i^4-2M_i^2(M_j^2+q^2)+(M_j^2-q^2)^2\right)\Bigg(A_1^{ij}(q^2)$$
$$-\frac{(M_i+M_j)^2\left(M_i^2-M_j^2-q^2\right)}{M_i^4-2M_i^2\left(M_j^2+q^2\right)+\left(M_j^2-q^2\right)^2}A_2^{ij}(q^2)\Bigg)^2\Bigg].$$

Form Factor Definitions

$$\langle V(p_V)|\bar{q}\gamma_\mu b|B(p_B)\rangle = \epsilon_{\mu\nu\rho\sigma}\epsilon^{*,\nu} p_B^\rho p_V^\sigma \, \frac{2V(q^2)}{m_B + m_V}$$

$$\langle V(p_V)|\bar{q}\gamma_\mu\gamma_5 b|B(p_B)\rangle = +i\epsilon^*_\mu(m_B + m_V)A_1(q^2) - i(p_B + p_V)_\mu(\epsilon^* \cdot q)\,\frac{A_2(q^2)}{m_B + m_V}$$

$$- iq_\mu(\epsilon^* q)\,\frac{2m_V}{q^2}\,\left(A_3(q^2) - A_0(q^2)\right)\;,$$

where the form factor combination $A_3(q^2)$ is defined by

$$A_3(q^2) = \frac{m_B + m_V}{2m_V}\,A_1(q^2) - \frac{m_B - m_V}{2m_V}\,A_2(q^2)$$

$$\langle V(p_V)|\bar{q}\sigma_{\mu\nu}q^\nu b|B(p_B)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} p_B^\rho p^\sigma \, 2T_1(q^2)$$

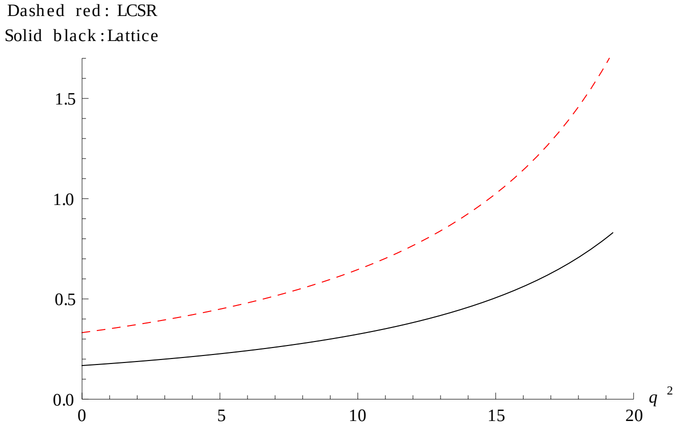
$$\langle V(p_V)|\bar{q}\sigma_{\mu\nu}q^\nu\gamma_5 b|B(p_B)\rangle = T_2(q^2)\left[\epsilon^*_\mu(m_B^2 - m_V^2) - (\epsilon^* \cdot q)\,(p_B + p)_\mu\right]$$

$$+ T_3(q^2)(\epsilon^* \cdot q)\left[q_\mu - \frac{q^2}{m_B^2 - m_V^2}\,(p_B + p)_\mu\right]\;,$$

Form Factor Relations

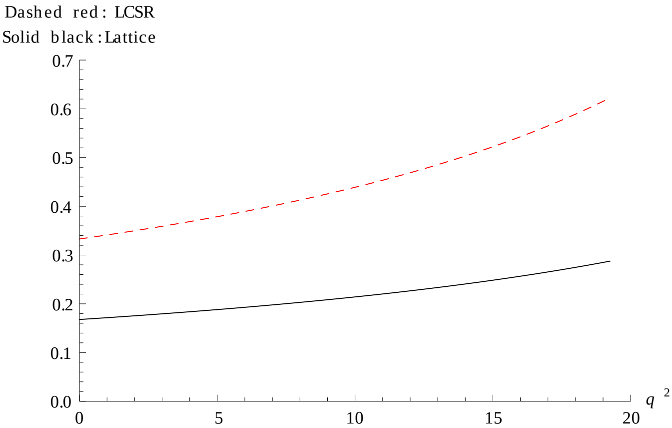
$$
\begin{aligned}
i\,f &= -A_1(M_B + M_V) \\
i\,g &= \frac{V}{M_B + M_V} \\
i\,h &= \frac{T_3}{M_B^2 - M_V^2} - \frac{T_1 - T_2}{q^2} \\
i\,g_+ &= -T_1 \\
i\,g_- &= \frac{\left(M_B^2 - M_V^2\right)(T_1 - T_2)}{q^2} \\
i\,a_+ &= \frac{A_2}{M_B + M_V} \\
i\,a_- &= \frac{A_1(M_B + M_V) - 2A_0 M_V - A_2(M_B - M_V)}{q^2}\,.
\end{aligned}
$$

$B \to K^*$ Tensor Form Factor Comparisson



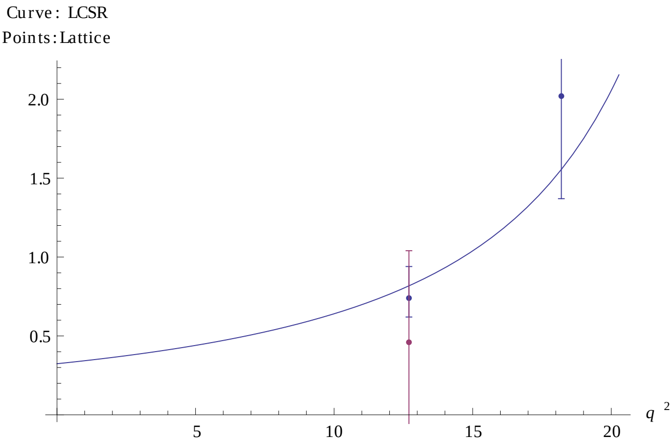
$T_1(q^2)$

$B \to K^*$ Tensor Form Factor Comparisson



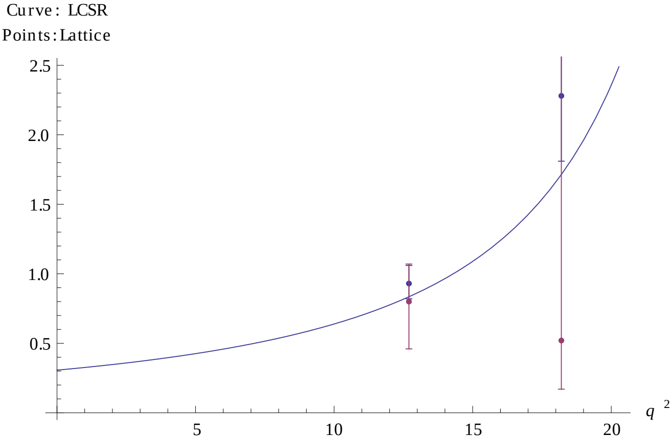
$T_2(q^2)$

$B \to \rho$ Form Factor Comparisson



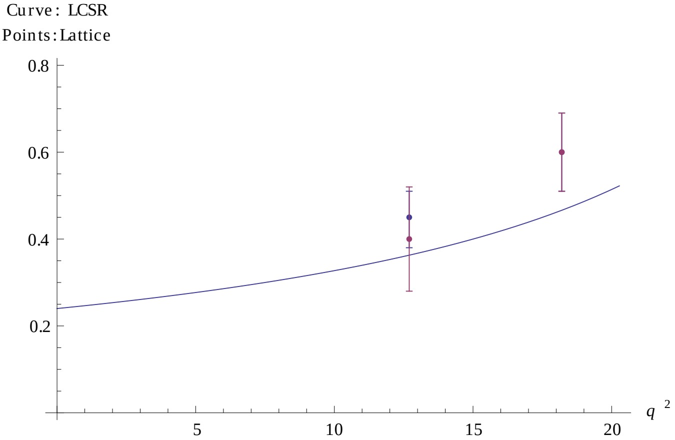
$V(q^2)$

$B \to \rho$ Form Factor Comparisson



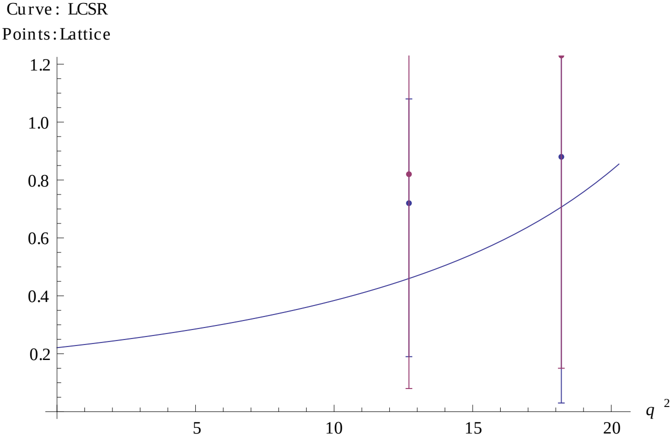
$A_0(q^2)$

$B \to \rho$ Form Factor Comparisson



$A_1(q^2)$

$B \to \rho$ Form Factor Comparisson



$A_2(q^2)$

Numerical Values

	$B \to K^* \ell^+ \ell^-$	$B \to \rho \ell \bar{\nu}_\ell$	$D \to \rho \ell \bar{\nu}_\ell$	$D \to K^* \ell \bar{\nu}_\ell$
q^2_{max} / GeV^2	19.25	20.28	1.20	0.96
w_{max}	3.04	3.47	1.41	1.29

	M_B	M_D	M_ρ	M_K^*	m_b	m_c
Value / GeV^2	5.279	1.87	0.776	0.892	4.2	1.17