Kaon Distribution Amplitude from QCD Sum Rules

A. Khodjamirian, Th. Mannel and M. Melcher

Theoretische Physik 1, Fachbereich Physik, Universität Siegen, D-57068 Siegen, Germany

Abstract

We present a new calculation of the first Gegenbauer moment $a_1^K$ of the kaon light-cone distribution amplitude. This moment is determined by the difference between the average momenta of strange and nonstrange valence quarks in the kaon. To calculate $a_1^K$, QCD sum rule for the diagonal correlation function of local and nonlocal axial-vector currents is used. Contributions of condensates up to dimension six are taken into account, including $O(\alpha_s)$-corrections to the quark-condensate term. We obtain $a_1^K = 0.05 \pm 0.02$, differing by the sign and magnitude from the recent sum-rule estimate from the nondiagonal correlation function of pseudoscalar and axial-vector currents. We argue that the nondiagonal sum rule is numerically not reliable. Furthermore, an independent indication for a positive $a_1^K$ is given, based on the matching of two different light-cone sum rules for the $K \to \pi$ form factor. With the new interval of $a_1^K$, we update our previous numerical predictions for SU(3)-violating effects in $B_{(s)} \to K$ form factors and charmless $B$ decays.
1 Introduction

Light-cone distribution amplitudes (DA’s) of hadrons are universal long-distance objects involved in many QCD approaches to exclusive hadronic processes with a large energy- or mass-scale. The variety of these processes (from form factors at large momentum transfers to heavy-meson exclusive decays), and the diversity of the approaches (from effective theories to QCD sum rules) makes the determination of DA’s an important task. In this paper we will mainly concentrate on the twist-2 DA of the kaon, defined by the standard expression

$$\left\langle K^- (q) \bar{s}(0) \gamma_\mu \gamma_5 [0, z] u(z) | 0 \right\rangle \bigg|_{z^2 \to 0} = -i q_\mu f_K \int_0^1 du \, e^{i u q \cdot z} \varphi_K (u, \mu) ,$$

where $\bar{u} = 1 - u$, $[0, z]$ is the path-ordered gauge-factor (Wilson line) and $\mu$ is the normalization scale determined by the interval $z^2$ near the light-cone.

The twist-2 DA $\varphi_K (u)$ is usually expanded in Gegenbauer polynomials

$$\varphi_K (u, \mu) = 6 u \bar{u} \left( 1 + \sum_{n=1}^\infty a^K_n (\mu) C^{3/2}_n (u - \bar{u}) \right) ,$$

with the multiplicatively renormalizable coefficients $a^K_n (\mu)$ (Gegenbauer moments). Taken at some low scale $\mu \sim 1$ GeV, the moments $a^K_1 (1$GeV) encode the long-distance dynamics. The anomalous dimension of $a^K_n$ grows with $n$. Hence, in many applications of DA’s where a high normalization scale is involved, the higher moments are suppressed, and only the lowest moments $a^K_{1,2}$ are retained.

As opposed to the pion case, where the odd moments $a^K_{1,3,..}$ vanish in the isospin symmetry limit, the first moment $a^K_1$ is expected to be as important as $a^K_2$. The nonzero value of $a^K_1$ reveals a flavour-SU(3) violation effect of $O(m_s - m_u,d)$. In physical terms, $a^K_1$ is proportional to the difference between the longitudinal momenta of the strange and nonstrange quark in the two-particle Fock component of the kaon. In our definition these fractions are $x_s = u$ and $x_{\bar{u}} = \bar{u}$, respectively, so that

$$\langle x_s - x_{\bar{u}} \rangle_{K^-} = \int_0^1 du \, \varphi_K (u) (u - \bar{u}) = \frac{3}{5} a^K_1 .$$

Hence, knowing or at least constraining $a^K_1 (1$GeV) is indispensable for predicting SU(3)-violation effects within any QCD approach that employs DA’s. The accurate knowledge of these effects is particularly important in $B$ decays to pions and kaons, in the context of CP-violation and CKM-matrix studies.

As originally suggested in [1], the few first Gegenbauer moments of DA’s can be calculated employing QCD sum rules [2] based on the local operator-product expansion (OPE) of a dedicated correlation function of two quark currents. To estimate $a^K_1$, two different correlation functions of kaon-interpolating quark currents have been considered in [1]:
the diagonal one, with two axial-vector currents and the nondiagonal one, with one pseudoscalar and one axial-vector current. In what follows we call also the sum rules obtained from the respective correlation functions diagonal and nondiagonal. In both cases there is an axial-vector current $\bar{s}\gamma_\mu\gamma_5 D_\alpha u$ containing a covariant derivative (where $D = \bar{D} - D$). The matrix element of this current entering the kaon term in the sum rule is proportional to $a^K_1$:

$$\langle K^- (q)|\bar{s}\gamma_\mu\gamma_5 i\bar{D}_\alpha u|0\rangle = -i q_\mu q_\alpha f_K\frac{3}{5} a^K_1.$$ (4)

This equation is easily obtained by expanding (1) near $z = 0$ and retaining the leading-twist term. In [1] only the leading-order contributions to OPE were taken into account and both sum rules predicted a positive value $[a^K_1(1\text{GeV})]_{CZ} \sim 0.17$. (5)

yielding $\langle x_s - x_u \rangle_{K^-} \simeq 0.10$. The positive sign of the SU(3)-breaking asymmetry is in accordance with an intuitive expectation for the kaon constituents: the heavier strange quark (antiquark) has a larger momentum fraction than the lighter nonstrange antiquark (quark). Although in contrast to intuition, also a negative sign of $a^K_1$ is possible, since the strange quark is not sufficiently heavy with respect to the typical hadronic scales, e.g. $m_s(1\text{ GeV}) \sim \Lambda_{QCD}$.

Recently, the nondiagonal QCD sum rule for $a^K_1$, together with the sum rules for the moments of $K^*$-meson DA’s, have been reconsidered and updated in [3]. The $O(\alpha_s)$ radiative corrections to the perturbative and quark-condensate terms in this sum rule have also been calculated. Importantly, the authors of [3] have traced a sign error in the leading perturbative term of the nondiagonal sum rule for $a^K_1$ written down in [1]. Correcting the sign and adding radiative corrections produce a drastic change in the hierarchy of terms in this sum rule. As a result, the value of $a^K_1$ has changed to $[a^K_1(1\text{GeV})]_{BB} = -0.18 \pm 0.09$. (6)

We have used the latter estimate in [4], in obtaining predictions on SU(3)-violation in exclusive $B$ decays and form factors.

As we already noted in [4], in order to get an independent estimate of $a^K_1$, one has to return to the diagonal correlation function and update the corresponding sum rule with the same $O(\alpha_s)$ accuracy. In this paper this task is to a large extent fulfilled. We undertake a new calculation of $a^K_1$ (and in parallel of $a^K_2$) from the diagonal correlation function keeping all terms up to dimension six in OPE and including the $O(\alpha_s)$-correction to the quark-condensate contribution. Quite surprisingly, our result

$$a^K_1(\mu \sim 1\text{GeV}) = 0.05 \pm 0.02,$$ (7)

substantially differs from the prediction (6) of the nondiagonal sum rule [3].

The paper is organized as follows. The details of the sum rule derivation are presented in Section 2, and the numerical analysis is performed in Section 3. In order to examine
the contradiction between our result and the one obtained in [3], in Section 4 we have a closer look at the nondiagonal sum rule. We find that it is rather difficult to obtain a reliable numerical estimate of $a^K_1$ from that sum rule. In section 5 we also give an additional independent argument in favour of positive $a^K_1$, employing light-cone sum rules for the spacelike $K \to \pi$ transition form factor. Furthermore, with the new estimate (7), in section 6 we update the numerical estimates for SU(3)-violating effects in $B$ decays obtained in [4]. Our conclusions are presented in section 7.

## 2 QCD sum rule for $a^K_n$

The way to derive QCD sum rules for the Gegenbauer coefficients is thoroughly explained in [1] and also in [3]. In the latter paper, instead of operators with covariant derivatives, an elegant device of nonlocal operators is employed, which we also use here. The underlying (diagonal) correlation function is chosen as

$$
\Pi(q^2, q \cdot z) = i \int d^4x \ e^{iq \cdot x} \langle 0 | T \{ \bar{u}(x) \not\!q \gamma_5 s(x), \bar{s}(0) \not\!q \gamma_5 [0, z] u(z) \} | 0 \rangle,
$$

(8)

where $z^2 = 0$ and we use an auxiliary nonlocal operator instead of a current with a fixed number of covariant derivatives. By inserting a complete set of hadronic states with $K$-meson quantum numbers in (8) we obtain

$$
\Pi^{hadr}(q^2, q \cdot z) = (q \cdot z)^2 \int_0^1 du \ e^{iuq \cdot z} \varphi_K(u)
$$

$$
+ \sum_{K_h} \frac{\langle 0 | \bar{u} \not\!q \gamma_5 s | K_h \rangle \langle K_h | \bar{s}(0) \not\!q \gamma_5 [0, z] u(z) | 0 \rangle}{m_{K_h}^2 - q^2},
$$

(9)

where the ground-state contribution of the kaon is shown explicitly and the sum takes into account the excited resonances and continuum states with the kaon quantum numbers. In the above we used (1) and the definition of the decay constant $\langle 0 | \bar{u} \gamma_\alpha \gamma_5 s | K^- (q) \rangle = if_K q_\alpha$. The raw sum rule is obtained by equating (9) to the result of OPE for $\Pi(q^2, q \cdot z)$ in terms of perturbative and condensate contributions. Below, the OPE result will be cast in the form

$$
\Pi^{OPE}(q^2, q \cdot z) = (q \cdot z)^2 \int_0^1 du \ e^{iuq \cdot z} \pi(u, q^2),
$$

(10)

where $\pi(u, q^2)$ may also contain delta function of $u$ and its derivatives. In addition we need the dispersion relation in $q^2$ for this function:

$$
\pi(u, q^2) = \frac{1}{\pi} \int_0^\infty ds \ \text{Im}_s \pi(u, s) \frac{s - q^2 - i\epsilon}{s - q^2 - i\epsilon},
$$

(11)

where subtraction terms are not essential, since they will vanish after Borel transformation. Employing quark-hadron duality, the sum over higher state contributions in (9) is then
approximated by the integral (11) where the lower limit is replaced by a certain effective threshold \( s_0^K \). Subtracting this integral (the continuum contribution) from both parts of the equation \( \Pi^{had} = \Pi^{OPE} \) and performing Borel transformation we obtain a generic sum rule for the DA:

\[
 f_K^2 \int_0^1 du \, e^{i\bar{u}q\cdot z} \varphi_K(u) e^{-m_{K}^2/M^2} = \int_0^1 du \, e^{i\bar{u}q\cdot z} \frac{1}{\pi} \int_0^{s_0^K} ds \, e^{-s/M^2} \operatorname{Im} \pi(u, s).
\] (12)

In order to project out the \( n \)-th moment and to obtain the sum rule for \( a_n^K \) one has to replace

\[
e^{i\bar{u}q\cdot z} \rightarrow C_n^{3/2}(u - \bar{u}),
\]

in both parts of this equation. The result is:

\[
a_n^K = \frac{2(2n + 3)}{3(n + 1)(n + 2)} \left( \frac{e^{m_{K}^2/M^2}}{f_K^2} \right) \frac{1}{\pi} \int_0^{s_0^K} ds \, e^{-s/M^2} \int_0^1 du \, C_n^{3/2}(u - \bar{u}) \operatorname{Im} \pi(u, s),
\] (13)

where the \( n \)-dependent factor comes from the normalization of Gegenbauer polynomials. At \( n = 0 \), \( a_0^K = 1 \) and (13) turns into the original SVZ sum rule for \( f_K^2 \) (see also [4] where the SU(3)-violation in this sum rule is investigated). In fact, only the few first moments of (13) are useful, practically \( n \leq 2 \), because as already realized in [1] the higher-dimensional condensate contributions rapidly grow with \( n \) and one cannot rely on local OPE [1]. A few comments are in order concerning the sum rule (13). First, the threshold parameter \( s_0^K \) generally depends on \( n \). Usually \( (s_0^K)_n \) is fitted together with \( a_n \) to achieve the maximal stability in the relevant region of Borel parameter where both higher-dimensional condensates and the continuum contribution (the duality estimate for higher states) are reasonably small, say, not exceeding 30% both. Second, the scale at which \( a_n \) is estimated from the QCD sum rules is of the order of \( M \), the characteristic virtuality of the correlation function. Since \( M \) covers an interval around 1 GeV it is just the scale we need.

The QCD calculation of the master function \( \pi(u, q^2) \) in a form of OPE is straightforward and we only give a few accompanying comments while explicitly representing the results. We expand the correlation function up to dimension 6, with the highest-dimension term in OPE coming from the four-quark condensate. The small parameter in the correlation function is the ratio \( m_s/M \), therefore the Wilson coefficients are also expanded in the strange quark mass. The \( u \)- and \( d \)-quark masses are put to zero. Generally, we neglect all terms where the power of \( m_s/M \) added to the dimension of the condensate exceeds 6. Furthermore, we use the Fock-Schwinger gauge for the vacuum gluon field, so that the gauge factor \([0, z] = 1\). The relevant contributions to the correlation function are listed below:

\[\text{We will not discuss an interesting possibility to introduce nonlocal condensates [5] which is a different approach.}\]
**Perturbative term:** As the calculation of this contribution given in the leading order by the loop diagram is not very complicated, we kept the full $m_s$ dependence. We get:

$$\pi^{\text{loop}}(u, q^2) = -\frac{3 u \bar{u}}{2 \pi^2} \log \left( \frac{m_s^2 - u q^2}{\mu^2} \right),$$

where $\mu$ is an arbitrary renormalization scale and the polynomial terms in $q^2$ are not shown. They, together with the $\mu$-dependent part vanish after Borel transformation. At present state, $O(\alpha_s)$ corrections to the diagonal correlation function are available only in the massless limit \([6, 5, 3]\). For $n = 0$ and at $O(m_s^0)$, the $\alpha_s$-correction to \((14)\) is less than 10%. In the sum rule for $a_1^K$, the leading-order perturbative contribution given by the convolution of \((14)\) with $C_1^{3/2}(2u - 1)$ starts at $O(m_s^2)$ and is numerically suppressed with respect to the quark-condensate term. For our calculation of $a_1$, we therefore assume that the $O(\alpha_s m_s^2)$ correction to the perturbative loop can be neglected. This conjecture can be verified in future with a direct calculation.

**Quark condensate:** At tree level, we get

$$\pi^{(\bar{q}q)}(u, q^2) = \frac{m_s \langle \bar{s}s \rangle}{(q^2)^2} \left[ \left( 1 - \frac{1}{3} \frac{m_s^2}{q^2} \right) \delta(u) - \frac{1}{3} \frac{m_s^2}{q^2} \delta'(u) \right],$$

where a generic notation $\langle \bar{q}q \rangle \equiv \langle 0 | \bar{q}q | 0 \rangle$, $q = u, s$, is used for the quark-condensate density. As \((14)\) is the dominant contribution, we also calculated the $O(\alpha_s)$-correction. This time we employed the Feynman gauge for the perturbative gluons, yielding additional contributions from the Wilson line. Since our main goal is the sum rule for $a_1^K$, we show here only the $n = 1$ moment of the correlation function:

$$\int_0^1 du \, C_1^{3/2}(u - \bar{u}) \pi^{\alpha_s(\bar{q}q)}(u, q^2) = \frac{\alpha_s C_F}{\pi} m_s \langle \bar{s}s \rangle \left[ 2 \left( \Delta - \log \left( \frac{q^2}{\mu^2} \right) \right) + \frac{31}{3} \right]$$

$$- \frac{\alpha_s C_F}{\pi} m_s \langle \bar{q}u \rangle \left( \frac{q^2}{(q^2)^2} \right),$$

where $\Delta = 2/(4 - D) - \gamma_E + \log 4\pi$. The corresponding expressions for $n = 0, 2, 3$ are given in appendix \(A\). Here, we neglected higher orders of $m_s$. The combination $m_s \langle \bar{s}s \rangle$ has zero anomalous dimension and the ultraviolet divergence is absorbed by the renormalization of $a_1^K$. Note that the contribution of the $u$-quark condensate absent in the leading order appears in $O(\alpha_s)$.

**Gluon condensate:** In calculating this contribution described by the quark-loop diagrams with an emission of two vacuum gluons, an additional term from the quark condensate has to be taken into account (see, e.g. \(I\) for details). We obtain:

$$\pi^{(G^2)}(u, q^2) = \frac{\langle G^2 \rangle}{12} \left[ \frac{m_s^2 u(2u q^2 + (1 - 3u)m_s^2)}{(m_s^2 - u q^2)^4} \right]$$

$$+ \frac{\delta(u)}{(q^2)^2} \left( 1 - \frac{m_s^2}{3 q^2} \right) + \frac{\delta(u)}{2(m_s^2 - q^2)^2} - \frac{\delta'(u) m_s^2}{3 (q^2)^3} \right],$$

\(17\),
where $\langle G^2 \rangle = \langle 0 \left| \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \right| 0 \rangle$ is the gluon-condensate density. Note that the terms in $O(m_s^0)$ cancel after convolution with odd Gegenbauer polynomials. In particular, for the $a^K_1$ sum rule the contribution of the gluon condensate term is proportional to $m_s^2$, in accordance with the SU(3)-symmetry limit and with the chirality structure of the correlation function.

**Quark-gluon condensate.** Here two effects contribute, as usual: the vacuum-gluon emission from the virtual quark line and the local expansion of the light-quark fields (up to three derivatives). The result reads:

$$\pi^{(sGs)}(u, q^2) = \frac{1}{3} m_s \langle sGs \rangle \delta'(u) \frac{1}{(q^2)^3},$$

where the notation $\langle sGs \rangle = \langle 0 \left| g_s \bar{s} \sigma_{\mu\nu} \frac{\lambda^a}{2} G^{\mu\nu} s \right| 0 \rangle$ for the quark-gluon condensate density is used. The contribution of the quark-gluon condensate with $u$ quarks vanishes in the $m_u = 0$ limit.

**Four-quark condensate:** For this contribution, we employ the usual vacuum saturation ansatz [2] factorizing all four-quark operators to the product of two quark condensates. The result reads:

$$\pi^{(qq)}(u, q^2) = - \frac{32 \pi \alpha_s}{81 (q^2)^3} \left( \langle ss \rangle^2 \left[ \delta(u) + \delta'(u) \right] + \langle \bar{u}u \rangle^2 \left[ \delta(\bar{u}) + \delta'(\bar{u}) \right] + \frac{9}{2} \langle ss \rangle \langle \bar{u}u \rangle \left[ \delta(u) + \delta(\bar{u}) \right] \right),$$

where the term proportional to $\langle \bar{u}u \rangle \langle ss \rangle$ vanishes for odd moments.

Summing up all contributions to $\pi(u, q^2)$ listed above, we use the obtained expression for the OPE of the correlation function to calculate various moments from (13). In particular, the desired sum rule for $a^K_1$ reads:

$$a^K_1 = \frac{e^{m_s^2/M^2}}{f_K^2} \left\{ \frac{5 m_s^4}{4 \pi^2} \int_{m_s^2}^{s_0^K} ds e^{-s/M^2} \frac{(m_s^2 - s)^2}{s^4} - \frac{5 m_s \langle ss \rangle}{3 M^2} \left( 1 + \frac{1}{2} \frac{m_s^2}{M^2} \right) \right\}$$

$$+ \left[ - \langle \bar{u}u \rangle + 2 \langle ss \rangle \left( \frac{25}{6} + \gamma_E - e^{-s_0^K / M^2} \frac{M^2}{s_0^K} - \log \left( \frac{M^2}{\mu^2} \right) - \text{Ei} \left( -s_0^K / M^2 \right) \right) \right]$$

$$+ \frac{5}{18 \pi M^4} \langle G^2 \rangle \left[ \gamma_E - \frac{1}{4} - \text{Ei} \left( -s_0^K / M^2 \right) + \log \left( \frac{m_s^2}{M^2} \right) + e^{-s_0^K / M^2} \left( \frac{M^4}{s_0^K} - \frac{M^2}{s_0^K} \right) \right]$$

$$+ \frac{5 m_s \langle sGs \rangle}{9 M^4} \frac{80 \alpha_s}{81 M^4} \left( \langle ss \rangle^2 - \langle \bar{u}u \rangle^2 \right),$$

where the perturbative term starts from $O(m_s^2)$:

$$\frac{5 m_s^4}{4 \pi^2} \int_{m_s^2}^{s_0^K} ds e^{-s/M^2} \frac{(m_s^2 - s)^2}{s^4} = \frac{5 m_s^2}{12 \pi^2} + O(m_s^4),$$
and \( \text{Ei}(x) = -\int_{-x}^{\infty} dt \ e^{-t/t} \). The \( O(m_s^2) \) loop contribution and the leading-order quark and quark-gluon condensate contributions to this sum rule have already been derived in \[1\], the other terms are new. Our result for \( \pi(u,q^2) \) allows to obtain also the even moments in \[13\]. We have compared them with the corresponding expressions in \[3\] (where the diagonal sum rule for \( a^K_n \) is only applicable for even \( n \)), and found agreement up to the terms of higher order in \( m_s \) which are not present in \[3\].

3 Numerical analysis

To analyze the sum rules numerically, we use the following intervals of the relevant input parameters: \( m_s(1 \text{ GeV}) = 130 \pm 20 \text{ MeV}, \langle \bar{q}q \rangle(1 \text{GeV}) = -(240 \pm 10 \text{ MeV})^3, \langle ss \rangle = (0.8 \pm 0.3)\langle \bar{q}q \rangle, \langle qGq \rangle(\mu) = (0.8 \pm 0.2 \text{ GeV}^2)\langle \bar{q}q \rangle(1 \text{ GeV}) \) (with a negligible scale-dependence) and \( \langle (\alpha_s/\pi)G^2 \rangle = 0.012 \pm 0.006 \text{ GeV}^4 \). Finally, we choose the renormalization scale to be \( \mu = M \) and adopt \( \alpha_s(M) \) with \( \tilde{\Lambda}_{QCD}^{(n_f=3)} = 320 \text{ MeV} \).

In order to check the input for the diagonal sum rule, we first analyze the \( n = 0 \) moment of \[13\], which yields the well-established SVZ sum rule for \( f_K \) \[2\]. We are now in a position to improve the accuracy of this sum rule adding the higher orders in \( m_s \) and \( O(\alpha_s\langle \bar{q}q \rangle) \) corrections. To this end, we fitted the threshold parameter \( s_0^K \) in order to achieve the maximal stability (weakest dependence on \( M^2 \)) within the optimal interval of the Borel parameter (the Borel window). The latter is the region of \( M^2 \) in which the OPE converges safely and excited states are suppressed. We found that for \( 0.5 \text{ GeV}^2 < M^2 < 1.0 \text{ GeV}^2 \) both the dimension-six four-quark condensate term and the subtracted continuum contribution in the sum rule are less than 30\%. In that region, the maximal stability is achieved for \( s_0^K = 1.05 \text{ GeV}^2 \). As a result, we get from the zeroth moment of \[13\] \( f_K = (0.92 \pm 0.02)f_K^{exp} \). This agreement adds more confidence in the validity of the lowest moments of the diagonal sum rule.

For \( a_1 \) the numerical prediction of the sum rule \[20\] is shown in Fig. 1(a). Here one has to move the Borel window to \( M^2 > 0.8 \text{ GeV}^2 \), in order to keep the OPE convergent. In the same region, the \( \alpha_s \)-correction to the condensate term is \(< 50\% \) of the zeroth order, so that one can trust the perturbative expansion. On the other hand, the continuum contribution is less than 10\% for \( M^2 \) up to 1.5 GeV. The contributions from the continuum and from the dimension-six term of the OPE are shown in figure \[1(b)\]. As the dependence of \( a_1 \) on the threshold is very weak (the only \( s_0^K \)-dependent contributions are suppressed by \( m_s^2/M^2 \) or \( \alpha_s \)), we use the value of \( s_0^K \) obtained from the sum rule for \( f_K \). Our confidence in the sum rule is also supported by the fact that the result for \( a_1^K \) is quite stable with respect to \( M \) within the Borel window (see figure \[1(a)\]). Finally we obtain for \( a_1^K \) the interval \[7\]. The estimated spread of our prediction is obtained by varying all input parameters within their allowed intervals (with \( 0.8 \text{ GeV}^2 < M^2 < 1.5 \text{ GeV}^2 \)) and summing up the resulting variations of \( a_1^K \) in quadratures. If we tried to reach certain cancellations and estimate \( a_1^K \) dividing \[20\] by the sum rule for \( f_K^2 \) (and not using the experimental value of \( f_K \)) the result would be practically the same.

Obtaining the interval \[7\], we tacitly assume the absence of large SU(3)-violating con-
Figure 1: (a) The Gegenbauer moment $a_1^K$ evaluated from the diagonal sum rule (20) as a function of the Borel parameter $M^2$ (the dotted lines indicate the uncertainties induced by the input parameters); (b) contributions from the dimension-six condensates (solid line) and from the continuum (dotted line), divided by the r.h.s. of (20).

Contributions originating from the higher-order terms of OPE (with dimension larger than 6) and not included in the diagonal sum rule. In fact, within the standard QCD sum rule approach there are two major sources of SU(3)-violation in OPE: the strange quark mass and the strange/nonstrange quark condensate ratio, all other effects, being derivatives of these two (assuming factorization of higher-dimensional condensates), are under control. An argument in favour of this conjecture is provided by the fact that one of the typical SU(3)-violating effects, the ratio $f_K/f_\pi$ is successfully reproduced [4] from the sum rule derived from the same diagonal correlation function with dimension-6 accuracy.

The fact that the continuum contribution to the sum rule (20) is small, indicates a relative suppression of hadronic states heavier than the kaon. The correlation function of axial-vector currents receives contributions from both pseudoscalar and axial-vector hadronic states. Importantly, the excited pseudoscalar resonances (the candidates for the lowest states [8] are $K(1460)$ and $K(1830)$) have decay constants proportional to light-quark masses and suppressed with respect to $f_K$. E.g., the sum rule estimates obtained in [9] yield $f_{K(1460),K(1830)} < 20$ MeV. Therefore, if one adds the contributions of excited $K$-resonances to the ground-state kaon term on l.h.s. of (12) (increasing the threshold $s^K_0$ correspondingly) one gets relative suppression factors $(f_{K(1460),K(1830)}/f_K)^2 < 2\%$ allowing to neglect these additional contributions altogether.

To investigate the impact of the axial-vector states we attempted to include explicitly the contribution from the lowest resonance $K_1(1270)$ [8] in the sum rule, in addition to the ground-state $K$-meson contribution. The threshold parameter $s^K_0$ is then correspondingly increased. We calculated the $K_1$ contribution from an independent QCD sum rule based on a correlation function where only the axial-vector states contribute. The procedure
is shown in more details in appendix B. Putting the result for $K_1$ into the hadronic representation of (8) shifts the central value for $a^K_1$ upwards by no more than 20%. From that we conclude that: 1) quark-hadron duality works reasonably well and 2) upon the inclusion of $K_1$, $a_1$ moves even further from negative values.

As a byproduct, we also calculated the second Gegenbauer moment of the kaon DA and obtain:

$$a^K_2(1 \text{ GeV}) = 0.27^{+0.37}_{-0.12}, \quad (21)$$

in agreement (within uncertainties) with the estimate of [3, 4] obtained from the same diagonal sum rule. Furthermore, putting $m_s \to 0$, $\langle \bar{s}s \rangle \to \langle \bar{q}q \rangle$, we also get an estimate for the second Gegenbauer moment of the pion:

$$a^\pi_2(1 \text{ GeV}) = 0.26^{+0.21}_{-0.09}, \quad (22)$$

in the ballpark of other current estimates of this parameter (see, e.g [12, 13]). Finally, we have checked that the moments (13) become unstable for $n > 2$ which is expected.

4 Is the nondiagonal sum rule reliable?

Our new result for $a^K_1(1 \text{ GeV})$ significantly differs from the estimate (6) obtained in [3] from the nondiagonal sum rule. The deviation remains even if we stretch both numbers towards each other by adding/subtracting the estimated uncertainties. The difference is also qualitative, because we predict a positive sign for the asymmetry $\langle x_s - x_{u,d} \rangle_K$. In fact, our $a^K_1$ has the same sign as (5) obtained in [1] from the diagonal sum rule. The fact that our estimate is smaller than (5), reflects the importance of the corrections taken into account in our calculation and absent in [1].

Let us take a closer look at the nondiagonal sum rule. Numerically evaluating the expressions presented in [3], we plot the result for $a^K_1$ as a function of $M^2$ in Fig. 2(a) where we have used the same input and estimated the uncertainties in the same way as for the sum rule (20). First of all, one notices that in this sum rule the three numerically large contributions: 1) the $O(\alpha_s^0)$ perturbative term, 2) the tree-level quark-condensate and 3) the quark-gluon condensate terms, almost cancel each other and this cancellation seems to happen accidentally. The remaining large contributions are the first-order in $\alpha_s$ terms in the perturbative and quark-condensate parts. Comparing $O(\alpha_s)$ and $O(\alpha_s^0)$ terms separately, we see that the former is about two times larger than the latter in the perturbative part. Both effects: the cancellation of the leading terms and enhancement of subleading terms cast doubt on the accuracy of the sum rule and on the validity of the perturbative expansion. Moreover, since the nondiagonal correlation function contains a pseudoscalar current, one has to worry about potentially important instanton effects, not taken into account in the local OPE series. This issue, however, deserves a separate study.

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2The value obtained in [1] from the nondiagonal sum rule is plagued by the error traced in [3] and should be ignored.

3We have found that also for $n = 0$, the $O(\alpha_s)$ term in the nondiagonal sum rule is larger than $O(\alpha_s^0)$ one.
Assessing the nondiagonal sum rule further, we notice that a Borel window for it practically does not exist. Indeed, as can be seen from Fig. 2(b), there is no region of $M^2$, in which both the convergence of the OPE, manifested by the smallness of the dimension six contribution (the term with $m_s$ multiplied by the quark-gluon condensate), and the suppression of excited states are guaranteed. In particular, $a^K_1$ is very sensitive to the choice of the threshold parameter. This may signal a greater role of higher states. On the other hand, only excited pseudoscalar $K$ resonances contribute to this sum rule, and as already noticed, their terms are suppressed with respect to the kaon ground-state term by the squares of the small decay constants. The situation is therefore somewhat controversial. In [3] the reliability of the nondiagonal sum rule is supported by obtaining $a^K_0 = 1.05 - 1.1$, close to 1. In fact, we repeated the numerical analysis of the nondiagonal sum rule for $a_0$ with our input parameters, and could not find a reasonable value for $s^K_0$ for which the Borel stability takes place. The value $s^K_0 = 1.8$ GeV$^2$ given in [3] seems to be somehow arbitrary. We conclude that the nondiagonal sum rule suffers from problems which are not yet fixed and therefore this sum rule is not reliable in its present form.

5 Constraining $a^K_1$ with light-cone sum rules for the $K \to \pi$ form factor

The form factors of pseudoscalar mesons at large momentum transfers are among the most important hadronic observables calculated using DA’s. Here we concentrate on the $K \to \pi$ transition form factor $f^{\pm}_{K\pi}$ defined via hadronic matrix element

$$\langle \pi^- (p - q) | \bar{s} \gamma_\mu u | K^0 (p) \rangle = 2 f^{+}_{K\pi} (q^2) p_\mu - (f^{+}_{K\pi} (q^2) - f^{-}_{K\pi} (q^2)) q_\mu .$$  \hspace{1cm} (23)
The form factor $f_{K^+}^{\pi}$ is measurable at small timelike $0 < q^2 < (m_K - m_\pi)^2$ in $K_{\ell 3}$ decays. However, it is completely legitimate to consider $f_{K^+}^{\pi}$ also at large spacelike $q^2 = -Q^2$ where it can be related to DA’s of pion and kaon. One well-known approach to calculate this form factor at large $Q^2$ is provided by the method of light-cone sum rules (LCSR) \[10\]. The calculation essentially repeats the applications of LCSR to the pion and kaon electromagnetic form factors \[11, 12\]. In particular, in \[12\] it was suggested to constrain the kaon DA by using two different LCSR for the same form factor. An illustrative calculation done in \[12\] at the twist 2 level, showed that at $a_1^K > 0$ two different LCSR agree. Here we repeat that analysis with a greater accuracy, taking into account also higher-twist effects in LCSR.

One starts from a generic correlation function

$T_{\mu\nu} = i \int d^4x e^{iqx} \langle 0 | \{ (\bar{q}_2(0)\gamma_\mu\gamma_5q_1(0)), (e_1q_1(x)\gamma_\nuq_1'(x) + e_2\bar{q}_2(x)\gamma_\nuq_2(x)) \} | P(p) \rangle$, (24)

where $q_{1,2}$ are the light-quark fields and $P = \pi$ or $K$. In both cases the correlation function is expanded at large $Q^2$ near light-cone $x^2 \sim 0$ up to twist-4 accuracy.

The first LCSR is then obtained from the invariant amplitude multiplying the $p_\mu p_\nu$ kinematical structure of the above correlation function by putting $q_1 = s$, $q_1' = u$, $q_2 = d$, $P = \pi^+$, $e_1 = 1$, $e_2 = 0$. The result involves only the pion DA’s starting from $\varphi_\pi(u)$ and is naturally independent of $a_1^K$. For simplicity, we give here only the leading-twist expression for the resulting LCSR \[12\]:

$f_{K^+}^{\pi}(Q^2) = \frac{f_\pi}{f_K} \int_{u_0^K}^1 du \varphi_\pi(u) e^{-\frac{aq^2}{um^2} + \frac{w_0^2}{M^2}}$, (25)

where $u_0^K = Q^2/(Q^2 + s_0^K)$ and $s_0^K$ is the duality threshold in the kaon channel, already determined above. The details of derivation and expressions for the higher twists can be found in \[12\].

The second LCSR comes from a different flavour pattern in \[24\]: $q_1 = d$, $q_2 = u$, $q_1' = s$, $e_1 = 0$, $e_2 = 1$, $P = K^0$. In this case the correlation function \[24\] reduces to a vacuum-to-kaon matrix element involving (after light-cone expansion) the kaon DA’s and containing $a_1^K$. Again, we only show the leading twist-two piece of the second LCSR:

$f_{K^+}^{\pi}(Q^2) = \frac{f_K}{f_\pi} \int_{u_0^K}^1 du \varphi_K(u) e^{-\frac{aq^2}{um^2} - \frac{u_0^2}{M^2}}$, (26)

where $u_0^n$ is related to the duality threshold $s_0^n$ in the pion channel by $s_0^n = (1 - u_0^n)(Q^2/u_0^n + m_\pi^2)$.

To evaluate the sum rules \[25\] and \[26\] numerically, we use the same input (and uncertainties) for the relevant DA parameters as in \[12\] (see also \[4\]), but leave $a_1^K$ as a free parameter. At $1$ GeV$^2 < Q^2 < 3$ GeV$^2$, one and the same observable $f_{K^+}^{\pi}(Q^2)$ is

\[\text{To avoid confusion, we note that in} \ [12] \ \text{an opposite sign convention for} \ a_1^K \ \text{is adopted.}\]
Figure 3: The ratio of two LCSR for $f_{K^+}^+$ defined as r.h.s. of (26)/r.h.s. of (25) at $a^K_1 = +0.05^{+0.02}_{-0.01}$ (left), $a^K_1 = 0$ (central) and $a^K_1 = -0.18 \pm 0.09$ [3] (right). The dashed lines indicate the uncertainties from varying all input parameters in the allowed ranges.

then calculated from LCSR in two different ways. This allows to constrain $a^K_1$ completely independent of the sum rule calculation done in previous sections.

We calculate the ratio of r.h.s. of two sum rules (25) and (26) as a function of $Q^2$. (Note that in this ratio some of uncertainties partially cancel). Ideally, the ratio should be equal to one. It is plotted in Fig. 3 for different values of $a^K_1$. The variation of the Borel parameter from 1 to 2 GeV$^2$ is included within the uncertainties. We find that for a positive value of $a^K_1$, the ratio is indeed consistent with 1. Remarkably, at $a^K_1 < 0$ it noticeably deviates from 1. Alternatively, one demands that the ratio of two LCSR is equal to 1 at, say, the middle value $Q^2 = 2$ GeV$^2$ and then solves it for $a^K_1$, yielding $a^K_1 \approx 0.2 \pm 0.1$, which is consistent with (7).

6 SU(3)-Violation in $B$ decays: an update

Having the new estimate (7) for the Gegenbauer moment $a^K_1$, we find it appropriate to give an update of SU(3)-violating effects in $B \to P$ form factors and $B \to PP$ decay amplitudes ($P = \pi, K$) calculated in [1] from LCSR, where we have used the interval (6) of $a^K_1$ from [3]. For consistency, in order to have correlated uncertainties, we also use the estimates (21) and (22) for $a^K_2$ and $a^\pi_2$. All other input parameters in LCSR are not changed.

Note that in the SU(3)-violating parts of LCSR, $a^K_1$ plays quite an important role. To demonstrate that, below we display the numerical predictions of LCSR (for the central values of parameters adopted in [1]) isolating the parts proportional to $a^K_1$:

$$f^+_{B \to K}(0) = (1.31 + 1.11 \cdot a^K_1(\mu_B)) f^+_{B \to \pi}(0),$$  
$$f^+_{B^* \to K}(0) = (1.25 - 1.02 \cdot a^K_1(\mu_B)) f^+_{B^* \to \pi}(0).$$  

(27)

In the above, the rest of SU(3)-violation is caused by differences between other parameters in the kaon and pion channel, mainly between $f_K$ and $f_\pi$, $s^K_0$ and $s^\pi_0$, $a^K_2$ and $a^\pi_2$. The SU(3)-violation in the parameters of higher-twist DA’s produce minor effects. Note that in (27) the characteristic scale of $\mu_B \sim O(m_B^2 - m_b^2)$ is used, and the scale-dependent
parameters, such as $a_1^K$ itself, have to be evolved to this scale using the known anomalous dimensions.

In obtaining the updates given below we follow the same procedure of estimating uncertainties as in [4], that is, the analytic expressions are used and all input parameters are varied in a correlated way.

First, we present the updated values of the $B \rightarrow \pi, K$ form factors which simply follow from (27) substituting our new estimate of $a_1^K$:

\[
\frac{f_{BK}^+(0)}{f_{B\pi}^+(0)} = 1.36^{+0.12}_{-0.09}, \quad \frac{f_{B\pi}^+(0)}{f_{B\pi}^+(0)} = 1.21^{+0.14}_{-0.11}.
\]

(28)

Note that these ratios differ mainly by the sign of the $a_1^K$ contribution and are therefore interchanged in their numerical value with respect to the ratios given in [4] after a sign change of $a_1^K$. Consequently, the pattern of $SU(3)$-violation in the factorizable part of the amplitudes also changes considerably:

\[
A_{fact}(B \rightarrow \pi K) = \frac{f_K}{f_\pi} A_{fact}(B \rightarrow \pi\pi) = 1.22
\]

\[
A_{fact}(B \rightarrow K\pi) = 1.36^{+0.12}_{-0.09}
\]

\[
A_{fact}(B \rightarrow K\bar{K}) = 1.65^{+0.14}_{-0.11}
\]

\[
A_{fact}(B_s \rightarrow K\bar{K}) = 1.52^{+0.18}_{-0.14}
\]

\[
A_{fact}(B_s \rightarrow K\pi) = 1.25^{+0.14}_{-0.12}
\]

In our notation of the factorizable amplitudes above the first (second) meson in the final state is the one containing the spectator quark of $B$ meson (the “emitted” one). Interestingly, the violation of the $SU(3)$-relation [14]

\[
A(B^- \rightarrow \pi^- \bar{K}^0) + \sqrt{2} A(B^- \rightarrow \pi^0 K^-) = \sqrt{2} \left( \frac{V_{us}}{V_{ud}} \right) A(B^- \rightarrow \pi^- \pi^0) \{1 + \delta_{SU(3)}\},
\]

(30)
determined by the parameter $\delta_{SU(3)}$ is robust with respect to $a_1^K$, and the prediction obtained in [4] (including nonfactorizable effects) remains nearly unchanged:

\[
\delta_{SU(3)} = (0.215^{+0.019}_{-0.010}) + (-0.009^{+0.009}_{-0.010})i.
\]

(31)

Further improvements in the input parameters in LCSR, in particular, a more precise determination of Gegenbauer moments will allow to decrease the uncertainties in (29) and (30).

### 7 Conclusions

In this paper we have reanalyzed the QCD sum rule prediction for the first Gegenbauer moment $a_1^K$ in the twist-2 DA of the kaon. This is an important $SU(3)$-violation effect.
reflecting the momentum asymmetry of strange and nonstrange quarks in the kaon. The advantage of the sum rule method is the ability to connect the value of $a_1^K$ directly to the fundamental QCD parameters: the mass of the $s$ quark and the difference between $s$- and $u,d$-quark condensates.

The diagonal correlation function provides a new estimate of $a_1^K$ which has the same positive sign as in the original calculation of [1] but a smaller value after including important perturbative correction to the quark condensate. We investigated the quality of this sum rule and found that all usual criteria are satisfied: Borel stability, hierarchy of power corrections, smallness of the higher-state contributions.

The OPE for the nondiagonal correlation function has been significantly improved in [3]. However, taking into account the numerical analysis presented in sect. 4, we think that the corresponding sum rule is numerically not safe for $a_1^K$ extraction. We therefore conclude that one should rely on the results from the diagonal sum rule.

Our estimate of $a_1^K$ has an uncertainty of $\sim 30\%$. It can be slightly improved further, e.g. calculating the $O(\alpha_s)$ correction to the (chirally-suppressed) perturbative part of the diagonal sum rule. However, the overall uncertainty will hardly become lower than 15-20%, due to the limited accuracy of the sum rule determination. Therefore, it would be very interesting to calculate $a_1^K$, that is, the matrix element (4) from lattice QCD.

**Note added**

After this paper was completed, the work [15] appeared, where a similar result for $a_1^K$ is obtained using a different method of operator identities.

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**A $\alpha_s\langle \bar{q}q \rangle$ for $n = 0, 2, 3$**

In addition to (16) we present here the $n = 0$ and $n = 2, 3$ projections of the $O(\alpha_s\langle \bar{q}q \rangle)$ contributions to the sum rule (13):

$$\int_0^1 du C_0^{3/2}(u - \bar{u}) \pi^{\alpha_s\langle \bar{q}q \rangle}(u, q^2) = -\frac{3 \alpha_s C_F m_s}{4 \pi} \frac{\bar{s}s}{(q^2)^2} - \frac{\alpha_s C_F m_s}{\pi} \frac{\bar{u}u}{(q^2)^2}$$

$$\int_0^1 du C_2^{3/2}(u - \bar{u}) \pi^{\alpha_s\langle \bar{q}q \rangle}(u, q^2) = \frac{\alpha_s C_F m_s}{\pi} \frac{\bar{s}s}{(q^2)^2} \left[ -\frac{25}{4} \left( \Delta - \log \left( -\frac{q^2}{\mu^2} \right) \right) - \frac{763}{24} \right]$$

$$- \frac{\alpha_s C_F}{2} \frac{7 m_s}{(q^2)^2} \bar{u}u,$$

$$\Delta \equiv \frac{\bar{u}u}{\frac{\alpha_s C_F}{2} m_s}.$$
\[ \int_0^1 du C_3^{3/2} \left( u - \bar{u} \right) \pi_{\alpha_s}^{\alpha_s(q\bar{q})}(u, q^2) = \frac{\alpha_s C_F m_s \langle \bar{s}s \rangle}{\pi} \left[ -\frac{157}{12} \left( \Delta - \log \left( -\frac{q^2}{\mu^2} \right) \right) + \frac{24629}{360} \right] \]

\[ -\frac{\alpha_s C_F 9 m_s \langle \bar{u}u \rangle}{2 (q^2)^2}, \] (34)

## B Including axial-vector mesons in the sum rule

In order to estimate the contribution from the axial-vector meson to the diagonal sum rule for \( a_1^K \), we need a slightly modified correlation function as compared to (33):

\[ \Pi_{\mu\nu}(q, z) = i \int d^4xe^{iq\cdot x} \langle 0| T \{ \bar{u}(x)\gamma_\mu\gamma_5 s(x), \bar{s}(0)\gamma_\nu\gamma_5 [0, z] u(z) \} |0\rangle, \] (35)

which can be decomposed in five invariant amplitudes:

\[ \Pi_{\mu\nu} = \Pi_1 q_\mu q_\nu + \Pi_2 g_{\mu\nu} + \Pi_3 q_\mu z_\nu + \Pi_4 z_\mu q_\nu + \Pi_5 z_\mu z_\nu. \] (36)

Note that the amplitude \( \Pi_1 \) above corresponds to \( \Pi \) from (33): \( (q\cdot z)^2 \Pi_1(q^2, q\cdot z) = \Pi(q^2, q\cdot z) \). For simplicity, we only take into account the lowest \( K_1(1270) \) state. The second axial-vector resonance \( K_1(1400) \) can also be included provided one adjusts the continuum threshold properly. But that will not noticeably change the effect of axial-vector states on the kaon contribution in the sum rule.

We proceed by defining the relevant hadronic matrix elements:

\[ \langle 0| \bar{u}\gamma_\mu\gamma_5 s|K_1(q, \lambda)\rangle = \epsilon_\mu^\lambda f_{K_1} m_{K_1}, \] (37)

\[ \langle K_1(q, \lambda)|\bar{s}(0)\gamma_\nu\gamma_5 [0, z] u(z)|0\rangle = \epsilon_\nu^\lambda f_{K_1} m_{K_1} \int_0^1 du e^{i\bar{u}qz} \varphi_{K_1}(u), \] (38)

where \( \varphi_{K_1}(u) \) is one of the DA’s of \( K_1(1270) \) and \( \epsilon \) is its polarization vector. Using the above definitions, one writes down the \( K_1 \) contribution to (33):

\[ \Pi^{K_1}_{\mu\nu}(q, z) = (q_\mu q_\nu - g_{\mu\nu} m_{K_1}^2) \frac{f_{K_1}^2}{m_{K_1}^2 - q^2} \int_0^1 du e^{i\bar{u}qz} \varphi_{K_1}(u). \] (39)

The kaon term is present only in \( \Pi_1 \), whereas \( K_1 \) contributes to both \( \Pi_1 \) and \( \Pi_2 \), and due to transversality, \( \epsilon \cdot q = 0 \), these two contributions are equal. Therefore, we can estimate the \( K_1 \) contribution from the sum rule for the invariant amplitude \( \Pi_2 \):

\[ -m_{K_1}^2 f_{K_1}^2 a_n^{K_1} e^{-m_{K_1}^2/M^2} = \frac{2(2n + 3)}{3(n + 1)(n + 2)} \frac{1}{\pi} \int_0^{K_1} ds e^{-s/M^2} \int_0^1 du C_n^{3/2}(u - \bar{u}) \text{Im}_s \pi_2(u, s). \] (40)

and use the estimate of \( f_{K_1}^2 a_n^{K_1} \) in the sum rule for \( \Pi_1 \). Since a rough estimate is sufficient for our purposes, we only include in \( \pi_2 \) the most important contributions of the perturbative
loop, quark condensate and quark-gluon condensate. We then turn to the sum rule for
\[ \Pi_1 = \int_0^1 du \, e^{i\theta q^2} \pi_1(u, q^2) \], where both the kaon and \( K_1 \) contribution are now included. After projecting out the \( n^{th} \) moment, we get:

\[
f_{K_1}^2 a_n^K e^{-m_{K_1}/M^2} + f_{K_1}^2 a_n^{K_1} e^{-m_{K_1}^2/M^2} = \frac{2(2n + 3)}{3(n + 1)(n + 2)} \pi \int_0^{s_0^{K_1}} ds \, e^{-s/M^2} \int_0^1 du \, C_n^{3/2}(u - \bar{u}) \text{Im}_{s} \pi_1(u, s). \tag{41}
\]

Here \( s_0^{K_1} \) is the new threshold, which naturally lies above \( m_{K_1}^2 \). Note that \( \pi_1(u, q^2) \) was already calculated for the sum rule (20).

Although the sum rules for \( \Pi_1 \) and \( \Pi_2 \) are different, we observe the best Borel stability of both by using the same duality threshold \( s_0^{K_1} \). As before, we first put \( n = 0 \) in (40) and choose \( s_0^{K_1} \) so that maximal Borel stability for \( f_{K_1}^2 \) is achieved. We get \( f_{K_1}^2 \approx 0.031^{+0.006}_{-0.003} \) GeV\(^2\) (in a good agreement with the experimental value of this decay constant extracted from the \( \tau \rightarrow K_1(1270)\nu_\tau \) partial width) and \( s_0^{K_1} \approx 1.7 \) GeV\(^2\). Furthermore, switching to \( n = 1 \) in (40) and dividing by \( f_{K_1}^2 \), we then obtain \( a_{K_1}^1 \approx -0.04^{+0.04}_{-0.03} \). Finally, our “\( K_1 \)-improved” value of \( a_{K_1}^1 \) obtained from (41) is

\[
a_{K_1}^1 = 0.07^{+0.02}_{-0.03} , \tag{42}
\]

slightly above the original one in (7).

References


