Theoretical Tools for Heavy Quark Physics

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Why Heavy Quark Physics?

- Flavour Mixing and CP-Violations are two of the most important topics of contemporary Particle Physics.
- It is all encoded in the UNITARY CKM MATRIX appearing in the charged current interaction:

\[
\mathcal{L} = \frac{g}{\sqrt{2}} (\bar{u} \, c \, t) \gamma^\mu (1 - \gamma_5) W^\mu \, V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}
\]

- Entries in the CKM matrix:

\[
V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}
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Heavy Quarks: $m_Q \gg \Lambda_{\text{QCD}}$

- Top Quark: $m_t \sim 175$ GeV (too heavy)
- Bottom Quark: $m_b \sim 4.5$ GeV (just o.k.)
- Charm Quark: $m_c \sim 1.5$ GeV (borderline case)
- Strange Quarks: $m_s \sim 0.1$ GeV (too light, but ...)

Almost all CKM matrix elements describe transitions involving one or even two heavy quarks.

Determination of these matrix elements involve to deal with the strong interaction of heavy quarks.
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CKM Matrix: Basics

- Tree dimensional (real) Rotation: Three angles $\theta_{ij}$

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- Single phase $\delta$:

$$U_{\delta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}.$$

- PDG CKM Parametrization:

$$V_{\text{CKM}} = U_{23}U_{\delta}^\dagger U_{13}U_{\delta}U_{12}$$

- Large Phases in $V_{ub} = |V_{ub}|e^{-i\gamma} = s_{13}e^{-i\delta_{13}}$ and $V_{td} = |V_{td}|e^{i\beta}$
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Out of six Unitarity Triangles only two have sides of comparable lengths:

- Depict the relation

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

as a triangle in the complex plane.
Effective Field Theories

- Weak decays:
  - Very different mass scales are involved:
    - $\Lambda_{\text{QCD}} \sim 200\text{ MeV}$: Scale of strong interactions
    - $m_c \sim 1.5\text{ GeV}$: Charm Quark Mass
    - $m_b \sim 4.5\text{ GeV}$: Bottom Quark Mass
    - $m_t \sim 175\text{ GeV}$ and $M_W \sim 81\text{ GeV}$:
      Top Quark Mass and Weak Boson Mass
    - $\Lambda_{\text{NP}}$: Scale of “new physics”
  
- At low scales the high mass particles / high energy degrees of freedom are irrelevant.
- Construct an “effective field theory” where the massive / energetic degrees of freedom are removed (“integrated out”)
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Integrating out heavy degrees of freedom

- $\phi$: light fields, $\Phi$: heavy fields with mass $\Lambda$
- Generating functional as a functional integral
  Integration over the heavy degrees of freedom

$$Z[j] = \int [d\phi][d\Phi] \exp \left( \int d^4x \left[ \mathcal{L}(\phi, \Phi) + j\phi \right] \right)$$

$$= \int [d\phi] \exp \left( \int d^4x \left[ \mathcal{L}_{\text{eff}}(\phi) + j\phi \right] \right) \quad \text{with}$$

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Theoretical Tools for Heavy Quark Physics
For length scales $x \gg 1/\Lambda$: local effective Lagrangian

Technically: \textit{(Operator Product) Expansion} in inverse powers of $\Lambda$

$$\mathcal{L}_{\text{eff}}(\phi) = \mathcal{L}_{\text{eff}}^{(4)}(\phi) + \frac{1}{\Lambda} \mathcal{L}_{\text{eff}}^{(5)}(\phi) + \frac{1}{\Lambda^2} \mathcal{L}_{\text{eff}}^{(6)}(\phi) + \cdots$$

- $\mathcal{L}_{\text{eff}}$ is in general non-renormalizable, but ...
- $\mathcal{L}_{\text{eff}}^{(4)}$ is the renormalizable piece
- For a fixed order in $1/\Lambda$: Only a finite number of insertions of $\mathcal{L}_{\text{eff}}^{(4)}$ is needed!
- $\rightarrow$ can be renormalized
- Renormalizability is not an issue here
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\mu = m_b \\
\mu = m_c \\
\mu = \Lambda_{QCD}
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Weak Gauge Bosons. Top Quark

Mass of the b Quark

Mass of the charm quark

Hadronic Scale

Renormalization

Renormalization

Renormalization

STOP
Heavy Quark Limit

- 1/$m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b, m_c \to \infty$ with fixed (four)velocity
  \[
  v_Q = \frac{p_Q}{m_Q}, \quad Q = b, c
  \]
- In this limit we have
  \[
  \begin{align*}
  m_{Hadron} &= m_Q \\
  p_{Hadron} &= p_Q
  \end{align*}
  \]
  \[
  v_{Hadron} = v_Q
  \]
- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics I!
**Heavy Quark Limit**

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  \end{cases}
  \quad v_{Hadron} = v_Q
  \]
- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics I!
Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

1/$m_Q$ Expansion: Substantial Theoretical Progress!

Static Limit: $m_b, m_c \to \infty$ with fixed (four)velocity

$$v_Q = \frac{p_Q}{m_Q}, \quad Q = b, c$$

In this limit we have

$$m_{Hadron} = m_Q$$
$$p_{Hadron} = p_Q$$

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\end{align*}$$

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- Static Limit: $m_b, m_c \to \infty$ with fixed (four)velocity
  \[
  v_Q = \frac{p_Q}{m_Q}, \quad Q = b, c
  \]
- In this limit we have
  \[
  \begin{align*}
  m_{\text{Hadron}} &= m_Q \\
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  \end{align*}
  \]
  \[
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  \]
- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics I!
The interaction of gluons is identical for all quarks.

Flavour enters QCD only through the mass terms:
- \( m \to 0 \): (Chiral) Flavour Symmetry (Isospin)
- \( m \to \infty \): Heavy Flavour Symmetry
- Consider \( b \) and \( c \) heavy: Heavy Flavour SU(2)

Coupling of the heavy quark spin to gluons:

\[
H_{int} = \frac{g}{2m_Q} \vec{Q} (\vec{\sigma} \cdot \vec{B}) Q \quad \xrightarrow{m_Q \to \infty} 0
\]

- Spin Rotations become a symmetry
- Heavy Quark Spin Symmetry: SU(2) Rotations

Spin Flavour Symmetry Multiplets
Heavy Quark Symmetries

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Heavy Quark Spin Symmetry: SU(2) Rotations

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Spin Rotations become a symmetry
Heavy Quark Spin Symmetry: SU(2) Rotations

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- Spin Rotations become a symmetry
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- Spin Flavour Symmetry Multiplets
# Mesonic Ground States

## Bottom:

\[
\begin{align*}
| (b\bar{u})_{J=0} \rangle &= |B^- \rangle \\
| (b\bar{d})_{J=0} \rangle &= |\bar{B}^0 \rangle \\
| (b\bar{s})_{J=0} \rangle &= |\bar{B}_s \rangle \\
| (b\bar{u})_{J=1} \rangle &= |B^{*-} \rangle \\
| (b\bar{d})_{J=1} \rangle &= |\bar{B}^{*-0} \rangle \\
| (b\bar{s})_{J=1} \rangle &= |\bar{B}_s^* \rangle
\end{align*}
\]

## Charm:

\[
\begin{align*}
| (c\bar{u})_{J=0} \rangle &= |D^0 \rangle \\
| (c\bar{d})_{J=0} \rangle &= |D^+ \rangle \\
| (c\bar{s})_{J=0} \rangle &= |D_s \rangle \\
| (c\bar{u})_{J=1} \rangle &= |D^{*0} \rangle \\
| (c\bar{d})_{J=1} \rangle &= |D^{*-} \rangle \\
| (c\bar{s})_{J=1} \rangle &= |D_s^* \rangle
\end{align*}
\]
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| (b\bar{u})_{J=1} \rangle &= | B^{*-} \rangle \\
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| (c\bar{s})_{J=1} \rangle &= | D^{*_s} \rangle
\end{align*}
\]
Mesonic Ground States

Bottom:

\[ |(b\bar{u})_{J=0}\rangle = |B^-\rangle \]
\[ |(b\bar{d})_{J=0}\rangle = |B^0\rangle \]
\[ |(b\bar{s})_{J=0}\rangle = |B_s^-\rangle \]
\[ |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle \]
\[ |(b\bar{d})_{J=1}\rangle = |B_{s*}^0\rangle \]
\[ |(b\bar{s})_{J=1}\rangle = |B_{s*}\rangle \]

Charm:

\[ |(c\bar{u})_{J=0}\rangle = |D^0\rangle \]
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\[ |(c\bar{u})_{J=1}\rangle = |D^{*0}\rangle \]
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\[ |(c\bar{s})_{J=1}\rangle = |D_{s}^*\rangle \]
### Mesonic Ground States

**Bottom:**

\[
\begin{align*}
\ket{(b\bar{u})_{J=0}} &= \ket{B^-} \\
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\ket{(b\bar{s})_{J=0}} &= \ket{B_s}
\end{align*}
\]

\[
\begin{align*}
\ket{(b\bar{u})_{J=1}} &= \ket{B^{*-}} \\
\ket{(b\bar{d})_{J=1}} &= \ket{B^{*0}} \\
\ket{(b\bar{s})_{J=1}} &= \ket{B_s^*}
\end{align*}
\]

**Charm:**

\[
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\end{align*}
\]

\[
\begin{align*}
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\ket{(c\bar{d})_{J=1}} &= \ket{D^{*+}} \\
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\end{align*}
\]
### Mesonic Ground States

**Bottom:**

$$\left| (b \bar{u})_{J=0} \right\rangle = \left| B^- \right\rangle \quad \left| (b \bar{u})_{J=1} \right\rangle = \left| B^{*-} \right\rangle$$

$$\left| (b \bar{d})_{J=0} \right\rangle = \left| B^0 \right\rangle \quad \left| (b \bar{d})_{J=1} \right\rangle = \left| B^{*0} \right\rangle$$

$$\left| (b \bar{s})_{J=0} \right\rangle = \left| B_s \right\rangle \quad \left| (b \bar{s})_{J=1} \right\rangle = \left| B_s^* \right\rangle$$

**Charm:**

$$\left| (c \bar{u})_{J=0} \right\rangle = \left| D^0 \right\rangle \quad \left| (c \bar{u})_{J=1} \right\rangle = \left| D^{*0} \right\rangle$$

$$\left| (c \bar{d})_{J=0} \right\rangle = \left| D^+ \right\rangle \quad \left| (c \bar{d})_{J=1} \right\rangle = \left| D^{*+} \right\rangle$$

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Mesonic Ground States

Bottom:

\[ |(b\bar{u})_{J=0}\rangle = |B^-\rangle \]
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\[ |(b\bar{s})_{J=0}\rangle = |B_s\rangle \]
\[ |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle \]
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Charm:

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\[
\begin{align*}
|b\bar{u}\rangle_{J=0} &= |B^-\rangle \\
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|b\bar{u}\rangle_{J=1} &= |B^{*-}\rangle \\
|b\bar{d}\rangle_{J=1} &= |\bar{B}^{*0}\rangle \\
|b\bar{s}\rangle_{J=1} &= |\bar{B}_s^*\rangle
\end{align*}
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Charm:

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\begin{align*}
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| (b \bar{u})_{J=0} \rangle &= | B^- \rangle \\
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| (c \bar{d})_{J=1} \rangle &= | D^{*+} \rangle \\
| (c \bar{s})_{J=1} \rangle &= | D^*_s \rangle
\end{align*}
\]
Baryonic Ground States

\[
\left| (ud)_0 Q \right\rangle_{1/2} = |\Lambda Q\rangle \\
\left| (uu)_1 Q \right\rangle_{1/2}, \left| (ud)_1 Q \right\rangle_{1/2}, \left| (dd)_1 Q \right\rangle_{1/2} = |\Sigma Q\rangle \\
\left| (uu)_1 Q \right\rangle_{3/2}, \left| (ud)_1 Q \right\rangle_{3/2}, \left| (dd)_1 Q \right\rangle_{3/2} = |\Sigma^* Q\rangle \\
\left| (us)_0 Q \right\rangle_{1/2}, \left| (ds)_0 Q \right\rangle_{1/2} = |\Xi Q\rangle \\
\left| (us)_1 Q \right\rangle_{1/2}, \left| (ds)_1 Q \right\rangle_{1/2} = |\Xi' Q\rangle \\
\left| (us)_1 Q \right\rangle_{3/2}, \left| (ds)_1 Q \right\rangle_{3/2} = |\Xi^* Q\rangle \\
\left| (ss)_1 Q \right\rangle_{1/2} = |\Omega Q\rangle, \left| (ss)_1 Q \right\rangle_{3/2} = |\Omega^* Q\rangle
\]
Baryonic Ground States

\[ [(ud)_{0/2}]_{1/2} = |\Lambda_Q\rangle \]

\[ [(uu)_{1/2}]_{1/2}, [(ud)_{1/2}]_{1/2}, [(dd)_{1/2}]_{1/2} = |\Sigma_Q\rangle \]

\[ [(uu)_{1/2}]_{3/2}, [(ud)_{1/2}]_{3/2}, [(dd)_{1/2}]_{3/2} = |\Sigma^*_Q\rangle \]

\[ [(us)_{0/2}]_{1/2}, [(ds)_{0/2}]_{1/2} = |\Xi_Q\rangle \]

\[ [(us)_{1/2}]_{1/2}, [(ds)_{1/2}]_{1/2} = |\Xi'_Q\rangle \]

\[ [(us)_{1/2}]_{3/2}, [(ds)_{1/2}]_{3/2} = |\Xi^*_Q\rangle \]

\[ [(ss)_{1/2}]_{1/2} = |\Omega_Q\rangle \]

\[ [(ss)_{1/2}]_{3/2} = |\Omega^*_Q\rangle \]
Baryonic Ground States

\[
\left| (ud)_0^Q \right\rangle_{1/2} = |\Lambda_Q\rangle \\
\left| (uu)_1^Q \right\rangle_{1/2}, \left| (ud)_1^Q \right\rangle_{1/2}, \left| (dd)_1^Q \right\rangle_{1/2} = |\Sigma_Q\rangle \\
\left| (uu)_1^Q \right\rangle_{3/2}, \left| (ud)_1^Q \right\rangle_{3/2}, \left| (dd)_1^Q \right\rangle_{3/2} = |\Sigma^*_Q\rangle \\
\left| (us)_0^Q \right\rangle_{1/2}, \left| (ds)_0^Q \right\rangle_{1/2} = |\Xi_Q\rangle \\
\left| (us)_1^Q \right\rangle_{1/2}, \left| (ds)_1^Q \right\rangle_{1/2} = |\Xi'_Q\rangle \\
\left| (us)_1^Q \right\rangle_{3/2}, \left| (ds)_1^Q \right\rangle_{3/2} = |\Xi^*_Q\rangle \\
\left| (ss)_1^Q \right\rangle_{1/2} = |\Omega_Q\rangle, \left| (ss)_1^Q \right\rangle_{3/2} = |\Omega^*_Q\rangle
\]
Baryonic Ground States

\[
\begin{align*}
[\text{(ud)}_0 Q]_{1/2} \rangle &= |\Lambda_Q\rangle \\
[\text{(uu)}_1 Q]_{1/2} \rangle, \quad [\text{(ud)}_1 Q]_{1/2} \rangle, \quad [\text{(dd)}_1 Q]_{1/2} \rangle &= |\Sigma_Q\rangle \\
[\text{(uu)}_1 Q]_{3/2} \rangle, \quad [\text{(ud)}_1 Q]_{3/2} \rangle, \quad [\text{(dd)}_1 Q]_{3/2} \rangle &= |\Sigma^*_Q\rangle \\
[\text{(us)}_0 Q]_{1/2} \rangle, \quad [\text{(ds)}_0 Q]_{1/2} \rangle &= |\Xi_Q\rangle \\
[\text{(us)}_1 Q]_{1/2} \rangle, \quad [\text{(ds)}_1 Q]_{1/2} \rangle &= |\Xi'_Q\rangle \\
[\text{(us)}_1 Q]_{3/2} \rangle, \quad [\text{(ds)}_1 Q]_{3/2} \rangle &= |\Xi^*_Q\rangle \\
[\text{(ss)}_1 Q]_{1/2} \rangle &= |\Omega_Q\rangle \\
[\text{(ss)}_1 Q]_{3/2} \rangle &= |\Omega^*_Q\rangle
\end{align*}
\]
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\[ [(ud)_0 Q]_{1/2} \rangle = |\Lambda_Q\rangle \]
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\[ [(uu)_1 Q]_{3/2} \rangle, [(ud)_1 Q]_{3/2} \rangle, [(dd)_1 Q]_{3/2} \rangle = |\Sigma^*_Q\rangle \]
\[ [(us)_0 Q]_{1/2} \rangle, [(ds)_0 Q]_{1/2} \rangle = |\Xi_Q\rangle \]
\[ [(us)_1 Q]_{1/2} \rangle, [(ds)_1 Q]_{1/2} \rangle = |\Xi'_Q\rangle \]
\[ [(us)_1 Q]_{3/2} \rangle, [(ds)_1 Q]_{3/2} \rangle = |\Xi^*_Q\rangle \]
\[ [(ss)_1 Q]_{1/2} \rangle = |\Omega_Q\rangle \]
\[ [(ss)_1 Q]_{3/2} \rangle = |\Omega^*_Q\rangle \]
Baryonic Ground States

\[
|{(ud)\,_{0Q}}_{1/2}\rangle = |\Lambda_Q\rangle \\
|{(uu)\,_{1Q}}_{1/2}\rangle, |{(ud)\,_{1Q}}_{1/2}\rangle, |{(dd)\,_{1Q}}_{1/2}\rangle = |\Sigma_Q\rangle \\
|{(uu)\,_{1Q}}_{3/2}\rangle, |{(ud)\,_{1Q}}_{3/2}\rangle, |{(dd)\,_{1Q}}_{3/2}\rangle = |\Sigma^*_Q\rangle \\
|{(us)\,_{0Q}}_{1/2}\rangle, |{(ds)\,_{0Q}}_{1/2}\rangle = |\Xi_Q\rangle \\
|{(us)\,_{1Q}}_{1/2}\rangle, |{(ds)\,_{1Q}}_{1/2}\rangle = |\Xi'_Q\rangle \\
|{(us)\,_{1Q}}_{3/2}\rangle, |{(ds)\,_{1Q}}_{3/2}\rangle = |\Xi^*_Q\rangle \\
|{(ss)\,_{1Q}}_{1/2}\rangle = |\Omega_Q\rangle, |{(ss)\,_{1Q}}_{3/2}\rangle = |\Omega^*_Q\rangle
\]
Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

\[ \langle H^{(*)}(v) | Q_v \Gamma Q_{v'} | H^{(*)}(v') \rangle = C_{\Gamma}(v, v') \xi(v \cdot v') \]

with \( H^{(*)}(v) = D^{(*)}(v) \) or \( B^{(*)}(v) \)

- \( C_{\Gamma}(v, v') \): Computable Clebsh Gordan Coefficient
- \( \xi(v \cdot v') \): Reduced Matrix Element
- \( \xi(v \cdot v') \): universal non-perturbative Form Faktor: Isgur Wise Funktion
- Normalization of \( \xi \) at \( v = v' \):

\[ \xi(v \cdot v' = 1) = 1 \]
Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

\[
\langle H^\ast(v) | Q_v \Gamma Q_{v'} | H^\ast(v') \rangle = C_{\Gamma}(v, v') \xi(v \cdot v')
\]

with \( H^\ast(v) = D^\ast(v) \) or \( B^\ast(v) \)

- \( C_{\Gamma}(v, v') \): Computable Clebsh Gordan Coefficient
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- Normalization of \( \xi \) at \( v = v' \):

\[
\xi(v \cdot v' = 1) = 1
\]
Wigner Eckart Theorem for HQS

HQS imply a “Wigner Eckart Theorem”

\[
\langle H^{(*)}(v) \mid Q_v \Gamma Q_{v'} \mid H^{(*)}(v') \rangle = C_{\Gamma}(v, v') \xi(v \cdot v')
\]

with \( H^{(*)}(v) = D^{(*)}(v) \) or \( B^{(*)}(v) \)

- \( C_{\Gamma}(v, v') \): Computable Clebsh Gordan Coefficient
- \( \xi(v \cdot v') \): Reduced Matrix Element
- \( \xi(v \cdot v') \): universal non-perturbative Form Faktor: Isgur Wise Funktion
- Normalization of \( \xi \) at \( v = v' \):

\[
\xi(v \cdot v' = 1) = 1
\]
HQS imply a "Wigner Eckart Theorem"

\[
\langle H^*(v) | Q_v \Gamma Q_{v'} | H^*(v') \rangle = C_{\Gamma}(v, v') \xi(v \cdot v')
\]

with \( H^*(v) = D^*(v) \) or \( B^*(v) \)

- \( C_{\Gamma}(v, v') \): Computable Clebsh Gordan Coefficient
- \( \xi(v \cdot v') \): Reduced Matrix Element
- \( \xi(v \cdot v') \): universal non-perturbative Form Faktor: Isgur Wise Funktion
- Normalization of \( \xi \) at \( v = v' \):

\[
\xi(v \cdot v' = 1) = 1
\]
Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

\[
\langle H^*(v) | Q_{v'} \Gamma Q_v | H^*(v') \rangle = C_\Gamma(v, v') \xi(v \cdot v')
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Normalization of \( \xi \) at \( v = v' \):

\[ \xi(v \cdot v' = 1) = 1 \]
The heavy mass limit can be formulated as an effective field theory. Define the static field $h_{\nu}$ for the velocity $\nu$:

$$h_{\nu}(x) = e^{im_{Q}\nu \cdot x} \frac{1}{2}(1 + \gamma) b(x)$$

HQET Lagrangian:

$$\mathcal{L} = \bar{h}_{\nu}(i\nu \cdot D)h_{\nu} + \frac{1}{2m_{Q}} \bar{h}_{\nu}(i\mathcal{D})^{2}h_{\nu} + \cdots$$

Dim-4 Term: Feynman rules, loops, renormalization...
The heavy mass limit can be formulated as an effective field theory.

Expansion in inverse powers of $m_Q$

Define the static field $h_v$ for the velocity $v$

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Heavy to Heavy: $B \rightarrow D\ell\bar{\nu}_\ell$ and $B \rightarrow D^*\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velocity $\nu$
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \rightarrow D^*\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_D^3 (\omega^2 - 1)^{1/2} P(\omega)(\mathcal{F}(\omega))^2$$

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- with $\omega = \nu\nu'$ and
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Thomas Mannel, University of Siegen
Theoretical Tools for Heavy Quark Physics
Heavy Quark Symmetries

- Normalization of the Form Factors is known at $\nu\nu' = 1$ from Heavy Quark Symmetries:
- Corrections can be calculated / estimated in HQET

$$F(\omega) = \eta_{\text{QED}}\eta_A \left[ 1 + \frac{\delta_1}{\mu^2} + \cdots \right] (\omega - 1)\rho^2 + O((\omega - 1)^2)$$

$$G(1) = \eta_{\text{QED}}\eta_V \left[ 1 + O \left( \frac{m_B - m_D}{m_B + m_D} \right) \right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_1/\mu^2 = -(8 \pm 4)\%$, $\eta_{\text{QED}} = 1.007$
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Unquenched Calculations become available!

- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

\[ F(1) = 0.91^{+0.03}_{-0.04} \]

\[ G(1) = 1.074 \pm 0.018 \pm 0.016 \]

A. Kronfeld et al.
Unquenched Calculations become available!

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A. Kronfeld et al.
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A. Kronfeld et al.
**Introduction / Motivation**

- Ancient Wisdom
- Inclusive Decays
- Recent Developments

**Heavy Quark Limit**

- Heavy Quark Symmetries
- Heavy Quark Effective Theory

---

**$B \rightarrow D^* \ell \bar{\nu}_\ell$**

- **ALEPH**
  - $32.6 \pm 2.0 \pm 1.3$
- **OPAL (partial reco)**
  - $37.8 \pm 1.2 \pm 2.3$
- **OPAL (excl)**
  - $37.9 \pm 1.6 \pm 1.6$
- **DELPHI (partial reco)**
  - $36.0 \pm 1.4 \pm 2.3$
- **BELLE**
  - $34.9 \pm 1.8 \pm 1.7$
- **CLEO**
  - $42.5 \pm 1.3 \pm 1.6$
- **BABBAR**
  - $34.4 \pm 0.3 \pm 1.2$
- **DELPHI (exclu)**
  - $37.6 \pm 1.7 \pm 1.9$

<table>
<thead>
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<th>$\chi^2$/dof</th>
<th>38.7/142</th>
</tr>
</thead>
</table>

**AVERAGE**

- $36.2 \pm 0.8$

---

**Theoretical Tools for Heavy Quark Physics**

- Thomas Mannel, University of Siegen
$B \rightarrow D \ell \bar{\nu}_\ell$

**ALEPH**
39.76 ± 10.00 ± 6.43

**CLEO**
45.12 ± 5.80 ± 3.51

**BELLE**
40.99 ± 4.40 ± 5.10

**Average**
42.60 ± 4.50

**HFAG**
Winter 2006

$\chi^2$/dof = 0.3/4

Thomas Mannel, University of Siegen

Theoretical Tools for Heavy Quark Physics
\[ V_{cb, excl} = (39.4 \pm 0.87^{+1.56}_{-1.24}) \times 10^{-3} \]

Bob Kowalewski @ ICHEP06

Possible Improvements:
More precise Lattice calculations
$V_{cb, excl} = (39.4 \pm 0.87^{+1.56}_{-1.24}) \times 10^{-3}$

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- Possible Improvements:
  More precise Lattice calculations
Inclusive Decays: Using OPE

Operator Product Expansion = Heavy Quark Expansion

\[
\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{\text{eff}} | B(v) \rangle|^2
\]

\[
= \int d^4 x \langle B(v) | \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}^\dagger(0) | B(v) \rangle
\]

\[
= 2 \text{Im} \int d^4 x \langle B(v) | T \{ \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}^\dagger(0) \} | B(v) \rangle
\]

\[
= 2 \text{Im} \int d^4 x e^{-im_b v \cdot x} \langle B(v) | T \{ \tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}_{\text{eff}}^\dagger(0) \} | B(v) \rangle
\]

- Last step: \( p_b = m_b v + k \),

Expansion in the residual momentum \( k \)
Inclusive Decays: Using OPE

**Operator Product Expansion = Heavy Quark Expansion**

(CHay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, M,...)

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= 2 \text{Im} \int d^4x \ e^{-im_b v \cdot x} \left\langle B(v) \mid T\{\tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}_{\text{eff}}^\dagger(0)\} \mid B(v) \right\rangle
\]

- **Last step:** $p_b = m_b v + k$
- **Expansion in the residual momentum $k$**
Inclusive Decays: Using OPE

Operator Product Expansion = Heavy Quark Expansion

\[ \Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) \left| \langle X | \mathcal{H}_{\text{eff}} | B(v) \rangle \right|^2 \]

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- Last step: \( p_b = m_b v + k \),

Expansion in the residual momentum \( k \)
Perform an OPE: $m_b$ is much larger than any scale appearing in the matrix element

$$
\int d^4 x e^{i m_b v x} T \{ \tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}^{\dagger}_{\text{eff}}(0) \} = \sum_{n=0}^{\infty} \left( \frac{1}{2 m_Q} \right)^n C_{n+3}(\mu) \mathcal{O}_{n+3}
$$

→ The rate for $B \rightarrow X_c \ell \bar{\nu}_\ell$ can be written as

$$
\Gamma = \Gamma_0 + \frac{1}{m_Q} \Gamma_1 + \frac{1}{m_Q^2} \Gamma_2 + \frac{1}{m_Q^3} \Gamma_3 + \cdots
$$

→ Perturbaton theory!

The $\Gamma_i$ are power series in $\alpha_s(m_Q)$:
Perform an OPE: $m_b$ is much larger than any scale appearing in the matrix element

$$\int d^4 x e^{i m_b v x} T\{\tilde{\mathcal{H}}_{\text{eff}}(x)\tilde{\mathcal{H}}_{\text{eff}}^\dagger(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

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→ Perturbaton theory!
\( \Gamma_0 \) is the decay of a free quark ("Parton Model")

\( \Gamma_1 \) vanishes due to Heavy Quark Symmetries

\( \Gamma_2 \) is expressed in terms of two parameters

\[
2M_H \mu_\pi^2 = - \langle H(v)|\bar{Q}_v(iD)^2Q_v|H(v)\rangle
\]

\[
2M_H \mu_G^2 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle
\]

\( \mu_\pi \): Kinetic energy and \( \mu_G \): Chromomagnetic moment

\( \Gamma_3 \) two more parameters

\[
2M_H \rho_D^3 = - \langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle
\]

\[
2M_H \rho_{LS}^3 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle
\]

\( \rho_D \): Darwin Term and \( \rho_{LS} \): Chromomagnetic moment

---

Thomas Mannel, University of Siegen

Theoretical Tools for Heavy Quark Physics
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- $\Gamma_0$ is the decay of a free quark ("Parton Model")
- $\Gamma_1$ vanishes due to Heavy Quark Symmetries
- $\Gamma_2$ is expressed in terms of two parameters

\[
2M_H \mu_\pi^2 = -\langle H(v) | \bar{Q}_v (iD)^2 Q_v | H(v) \rangle
\]
\[
2M_H \mu_G^2 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu)(iD^\nu) Q_v | H(v) \rangle
\]

$\mu_\pi$: Kinetic energy and $\mu_G$: Chromomagnetic moment

- $\Gamma_3$ two more parameters

\[
2M_H \rho_D^3 = -\langle H(v) | \bar{Q}_v (iD_\mu)(ivD)(iD^\mu) Q_v | H(v) \rangle
\]
\[
2M_H \rho_{LS}^3 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu)(ivD)Q_v | H(v) \rangle
\]

$\rho_D$: Darwin Term and $\rho_{LS}$: Chromomagnetic moment
New: $1/m_b^4$ Contribution $\Gamma_4$ (Dassinger, Turczyk, M.)

- Five new parameters:
  
  $\langle \vec{E}^2 \rangle$: Chromoelectric Field squared
  $\langle \vec{B}^2 \rangle$: Chromomagnetic Field squared
  $\langle (\vec{p}^2)^2 \rangle$: Fourth power of the residual $b$ quark momentum
  $\langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle$: Mixed Chromomag. Mom. and res. Momentum
  $\langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle$: Mixed Chromomag. field and res. helicity

- Some of these can be estimated in naive factorization
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Heavy to Heavy: $B \to X_c \ell \bar{\nu}_\ell$

- Determine the HQE parameters from
  - Charged lepton energy spectrum
  - Hadronic invariant mass spectrum
- From the theoretical side:
  Calculation of moments of the spectra

\[
\langle M^n_X \rangle = \frac{1}{\Gamma} \int dM_X M^n_X \int_{E_{\text{cut}}} dE_\ell \frac{d^2\Gamma}{dM_X dE_\ell}
\]

\[
\langle E^n_\ell \rangle = \frac{1}{\Gamma} \int dM_X \int_{E_{\text{cut}}} dE_\ell E^n_\ell \frac{d^2\Gamma}{dM_X dE_\ell}
\]
**Heavy to Heavy:** \[ B \rightarrow X_c \ell \bar{\nu}_\ell \]

- Determine the HQE parameters from
  - Charged lepton energy spectrum
  - Hadronic invariant mass spectrum

- From the theoretical side:
  **Calculation of moments of the spectra**

\[
\langle M_X^n \rangle = \frac{1}{\Gamma} \int dM_X M_X^n \int_{E_{cut}} dE_\ell \frac{d^2\Gamma}{dM_X dE_\ell}
\]

\[
\langle E_\ell^n \rangle = \frac{1}{\Gamma} \int dM_X \int_{E_{cut}} dE_\ell E_\ell^n \frac{d^2\Gamma}{dM_X dE_\ell}
\]

---

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Theoretical Tools for Heavy Quark Physics
Heavy to Heavy: $B \rightarrow X_c \ell \bar{\nu}_\ell$

- Determine the HQE parameters from
  - Charged lepton energy spectrum
  - Hadronic invariant mass spectrum

- From the theoretical side:
  Calculation of moments of the spectra

\[
\langle M^n_X \rangle = \frac{1}{\Gamma} \int dM_X \, M^n_X \int_{E_{\text{cut}}} dE_\ell \, \frac{d^2\Gamma}{dM_X \, dE_\ell} \\
\langle E^n_\ell \rangle = \frac{1}{\Gamma} \int dM_X \int_{E_{\text{cut}}} dE_\ell \, E^n_\ell \frac{d^2\Gamma}{dM_X \, dE_\ell}
\]
Hadronic Invariant Mass Moments (Buchmüller, Flächer)

\( \langle M_X \rangle \) (GeV)

\( \langle M_X^2 \rangle \) (GeV\(^2\))

\( \langle M_X^3 \rangle \) (GeV\(^3\))

\( \langle (M_X^2 - \langle M_X \rangle)^2 \rangle \) (GeV\(^4\))

\( E_{\text{cut}} \) (GeV)
Lepton Energy Moments I (Buchmüller, Flächer)

(a) BR

(b) $<E_L>$ (GeV)

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Theoretical Tools for Heavy Quark Physics
Lepton Energy Moments II  
(Buchmüller, Flächer)
\[ V_{cb,incl} = \left( 41.96 \pm 0.23_{\text{exp}} \pm 0.35_{\text{HQE}} \pm 0.59_{\Gamma_{sl}} \right) \times 10^{-3} \]

O. Buchmüller, HQL2006
Calculation of spectra within the OPE

\[
\frac{d\Gamma}{dy} \propto \Theta(1 - y - \rho) \left[ 2 - \frac{\mu^2}{\pi^2} \left( \frac{\rho}{1 - \rho} \right)^2 \left\{ 3 - 4 \left( \frac{\rho}{1 - \rho} \right) \right\} \right]
\]

- \( y = \frac{2E_\ell}{m_b} \)
- \( \rho = \frac{m_c^2}{m_b^2} \)
In the massless case this becomes for \( B \rightarrow X_u \ell \bar{\nu}_\ell \)

\[
\frac{d\Gamma}{dy} \xrightarrow{y \rightarrow 1} \frac{G_F^2 |V_{ub}|^2 m_b^5}{96\pi^3} \left[ \Theta(1 - y) + \frac{\mu_\pi^2 - \mu_G^2}{6m_b^2} \delta(1 - y) + \frac{\mu_\pi^2}{6m_b^2} \delta'(1 - y) + \cdots \right]
\]

Likewise for \( B \rightarrow X_s \gamma \) (\( x = \frac{2E_\gamma}{m_b} \))

\[
\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb}^*|^2 |C_7|^2 \left( \delta(1 - x) - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \delta'(1 - x) + \frac{\mu_\pi^2}{6m_b^2} \delta''(1 - x) + \cdots \right)
\]
\[ \frac{d\Gamma}{dy} \bigg|_{y=1} = \frac{G_F^2 |V_{ub}|^2 m_b^5}{96\pi^3} \]
\[ \left[ \Theta(1 - y) + \frac{\mu_\pi^2 - \mu_G^2}{6m_b^2} \delta(1 - y) + \frac{\mu_\pi^2}{6m_b^2} \delta'(1 - y) + \cdots \right] \]

Likewise for \( B \rightarrow X_s \gamma \) (\( x = \frac{2E_\gamma}{m_b} \))

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\[ \left( \delta(1 - x) - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \delta'(1 - x) + \frac{\mu_\pi^2}{6m_b^2} \delta''(1 - x) + \cdots \right) \]
Resummation into a shape function or light cone distribution function (Bigi, Shifman, Uraltsev, Neubert, M., ...)

\[ 2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))|B(v)\rangle \]

such that

\[
\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}|^2 m_b^5}{96\pi^3} \int d\omega \Theta(m_b(1 - y) - \omega) f(\omega)
\]

and

\[
\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2 f(m_b[1 - x])
\]
Shape- or Light-Cone Distribution Functions

- Resummation into a shape function or light cone distribution function (Bigi, Shifman, Uraltsev, Neubert, M., ...)

\[ 2M_B f(\omega) = \langle B(v) | \bar{b}_v \delta(\omega + i(n \cdot D)) | B(v) \rangle \]

such that

\[ \frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}|^2 m_b^5}{96\pi^3} \int d\omega \Theta(m_b(1 - y) - \omega) f(\omega) \]

and

\[ \frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb}^*|^2 |C_7|^2 f(m_b[1 - x]) \]
General Structure:

\[
\frac{d\Gamma}{dx} = \Gamma_0 \left[ \sum_i a_i \left( \frac{1}{m_b} \right)^i \delta^{(i)} (1 - x) + \mathcal{O}\left(\frac{1}{m_b} \right)^{i+1} \delta^{(i)} (1 - x) \right]
\]

Coefficients \(a_i\) are the moments of the spectrum:

Moment Expansion of \(f\) in terms of HQE parameters:

\[
f(\omega) = \delta(\omega) + \frac{\mu^2}{6m_b^2} \delta^{''}(\omega) - \frac{\rho_D^3}{18m_b^3} \delta^{'''}(\omega) + \cdots
\]

Twist Expansion in complete analogy to deep inelastic scattering
General Structure:

$$\frac{d\Gamma}{dx} = \Gamma_0 \left[ \sum_i a_i \left( \frac{1}{m_b} \right)^i \delta^{(i)}(1-x) + \mathcal{O}\left((1/m_b)^{i+1}\delta^{(i)}(1-x)\right) \right]$$

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Twist Expansion in complete analogy to deep inelastic scattering
General Structure:

\[
\frac{d\Gamma}{dx} = \Gamma_0 \left[ \sum_i a_i \left( \frac{1}{m_b} \right)^i \delta^{(i)}(1 - x) + O((1/m_b)^{i+1} \delta^{(i)}(1 - x)) \right]
\]

Coefficients \( a_i \) are the moments of the spectrum:

- Moment Expansion of \( f \) in terms of HQE parameters:

\[
f(\omega) = \delta(\omega) + \frac{\mu^2}{6m_b^2} \delta''(\omega) - \frac{\rho_D^3}{18m_b^3} \delta'''(\omega) + \cdots
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Twist Expansion in complete analogy to deep inelastic scattering
Problem: How to calculate corrections to the shape functions?

More than two scales involved!

Inclusive Rates in the Endpoint become \((\text{Korchemski, Sterman})\)

\[ d\Gamma = H \ast J \ast S \]

\(d\Gamma\): Soft Collinear Effective Theory

- \(H\): Hard Coefficient Function, Scales \(\mathcal{O}(m_b)\)
- \(J\): Jet Function, Scales \(\mathcal{O}(\sqrt{m_b\Lambda_{QCD}})\)
- \(S\): Shape function, Scales \(\mathcal{O}(\Lambda_{QCD})\)
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- **S**: Shape function, Scales \(\mathcal{O}(\Lambda_{\text{QCD}})\)
Problem: How to calculate corrections to the shape functions?

More than two scales involved!

Inclusive Rates in the Endpoint become (Korchemski, Sterman)

\[ d\Gamma = H * J * S \]

with * = Convolution

- \( H \): Hard Coefficient Function, Scales \( \mathcal{O}(m_b) \)
- \( J \): Jet Function, Scales \( \mathcal{O}(\sqrt{m_b}\Lambda_{QCD}) \)
- \( S \): Shape function, Scales \( \mathcal{O}(\Lambda_{QCD}) \)
Problem: How to calculate corrections to the shape functions?

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Basics of Soft Collinear Effective Theory

- Heavy-to-light decays:
  Kinematic Situations with energetic light quarks hadronizing into jets or energetic light mesons

\[ p_{\text{fin}}: \text{Momentum of a light final state meson} \]

\[ p_{\text{fin}}^2 \sim O(\Lambda_{\text{QCD}} m_b) \quad v \cdot p_{\text{fin}} \sim O(m_b) \]

- Use light-cone vectors \( n^2 = \bar{n}^2 = 0, \, n \cdot \bar{n} = 2: \)

\[ p_{\text{fin}} = \frac{1}{2}(n \cdot p_{\text{fin}}) \bar{n} \quad \text{and} \quad v = \frac{1}{2}(n + \bar{n}) \]

- Momentum of a light quark in such a meson:

\[ p_{\text{light}} = \frac{1}{2}[(n \cdot p_{\text{light}}) \bar{n} + (\bar{n} \cdot p_{\text{light}}) n] + p_{\text{light}}^\perp \]
Basics of Soft Collinear Effective Theory

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Basics of Soft Collinear Effective Theory

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Thomas Mannel, University of Siegen

Theoretical Tools for Heavy Quark Physics
Basics of Soft Collinear Effective Theory

- **Heavy-to-light decays:** Kinematic Situations with energetic light quarks hadronizing into jets or energetic light mesons
  - $p_{\text{fin}}$: Momentum of a light final state meson
    \[
    p_{\text{fin}}^2 \sim \mathcal{O}(\Lambda_{\text{QCD}} m_b) \quad \mathbf{v} \cdot p_{\text{fin}} \sim \mathcal{O}(m_b)
    \]
  - Use light-cone vectors $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$:
    \[
    p_{\text{fin}} = \frac{1}{2} (n \cdot p_{\text{fin}}) \bar{n} \quad \text{and} \quad \mathbf{v} = \frac{1}{2} (n + \bar{n})
    \]
- Momentum of a light quark in such a meson:
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  p_{\text{light}} = \frac{1}{2} [(n \cdot p_{\text{light}}) \bar{n} + (\bar{n} \cdot p_{\text{light}}) n] + p_{\text{light}}^\perp
  \]
SCET Power Counting

- Define the parameter \( \lambda = \sqrt{\frac{\Lambda_{\text{QCD}}}{m_b}} \)
- The light quark invariant mass (or virtuality) is assumed to be
  \[
  p_{\text{light}}^2 = (n \cdot p_{\text{light}})(\bar{n} \cdot p_{\text{light}}) + (p_{\text{light}}^\perp)^2 \sim \lambda^2 m_b^2
  \]
- The components of the quark momentum have to scale as
  \[
  (n \cdot p_{\text{light}}) \sim m_b \quad (\bar{n} \cdot p_{\text{light}}) \sim \lambda^2 m_b \quad p_{\text{light}}^\perp \sim \lambda m_b
  \]
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A brief look at SCET

(Bauer, Stewart, Pirjol, Beneke, Feldmann ...)

- QCD quark field $Q$ is split into a collinear component $\xi$ and a soft one with $\xi = \frac{1}{4} \not{n} \not{h} q$
- The Lagrangian $\mathcal{L}_{\text{QCD}} = \bar{q}(i\not{D})q$ is rewritten in terms of the collinear field

$$\mathcal{L} = \frac{1}{2} \bar{\xi} \not{h}_{+}(in_{-}D)\xi - \bar{\xi}i\not{D}_{\perp} \frac{1}{i\not{n} + D + i\epsilon} \not{h}_{+}i\not{D}_{\perp}\xi$$

- Expansion according to the above power counting:

$$in_{+}D = in_{+}\partial + gn_{+}A_{c} + gn_{+}A_{us} = in_{+}D_{c} + gn_{+}A_{us}$$

- Leading $\mathcal{L}$ becomes non-local: Wilson lines
A brief look at SCET  

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\[
\mathcal{L} = \frac{1}{2} \bar{\xi} \slashed{n} (in_+ D) \xi - \bar{\xi} i\slashed{D}_\perp \frac{1}{in_+ D + i \epsilon} \frac{1}{2} \slashed{n} + i\slashed{D}_\perp \xi
\]

- Expansion according to the above power counting:

\[
in_+ D = in_+ \partial + gn_+ A_c + gn_+ A_{us} = in_+ D_c + gn_+ A_{us}
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$$\mathcal{L} = \frac{1}{2} \bar{\xi} \not{\hat{n}}_+ (in_- D)\xi - \bar{\xi} i\not{D}_\perp \frac{1}{in_+ D + i\epsilon} \frac{\hat{n}_+}{2} i\not{D}_\perp \xi$$

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$$\mathcal{L} = \frac{1}{2} \bar{\xi} \hat{\eta}_+(i\not{n} - \not{D})\xi - \bar{\xi}i\not{D} \frac{1}{in_+D + i\epsilon} \frac{1}{2} \hat{\eta}_+ i\not{D} \perp \xi$$

- Expansion according to the above power counting:

$$in_+D = in_+\partial + gn_+A_c + gn_+A_{us} = in_+D_c + gn_+A_{us}$$

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- QCD quark field $Q$ is split into a collinear component $\xi$ and a soft one with $\xi = \frac{1}{4} \not{n} - \not{q}$

- The Lagrangian $\mathcal{L}_{\text{QCD}} = \bar{q}(i\not{D})q$ is rewritten in terms of the collinear field

$$\mathcal{L} = \frac{1}{2} \bar{\xi} \not{D} (in_- D) \xi - \bar{\xi} i\not{D} \frac{1}{in_+ D + i\epsilon} 2 i\not{D} \perp \xi$$

- Expansion according to the above power counting:

$$in_+ D = in_+ \partial + gn_+ A_c + gn_+ A_{\text{us}} = in_+ D_c + gn_+ A_{\text{us}}$$

- Leading $\mathcal{L}$ becomes non-local: Wilson lines
Allows us to calculate radiative corrections systematically

- Extremely important for the determination of $V_{ub}$
- Shape functions are modelled or taken from $B \rightarrow X_{s\gamma}$

$$V_{ub, incl} = (4.48 \pm 0.20_{exp} \pm 0.27_{m_{b, theo}}) \times 10^{-3}$$
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Results for $V_{ub}$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO (endpoint)</td>
<td>4.09</td>
<td>± 0.48</td>
<td>± 0.36</td>
</tr>
<tr>
<td>BELLE (endpoint)</td>
<td>4.82</td>
<td>± 0.45</td>
<td>± 0.30</td>
</tr>
<tr>
<td>BABAR (endpoint)</td>
<td>4.39</td>
<td>± 0.25</td>
<td>± 0.39</td>
</tr>
<tr>
<td>BABAR ($E_{\nu}, q^2$)</td>
<td>4.57</td>
<td>± 0.31</td>
<td>± 0.41</td>
</tr>
<tr>
<td>BELLE $m_x$</td>
<td>4.06</td>
<td>± 0.27</td>
<td>± 0.24</td>
</tr>
<tr>
<td>BELLE sim. ann. ($m_x, q^2$)</td>
<td>4.37</td>
<td>± 0.46</td>
<td>± 0.29</td>
</tr>
<tr>
<td>BABAR ($m_x, q^2$)</td>
<td>4.75</td>
<td>± 0.35</td>
<td>± 0.32</td>
</tr>
<tr>
<td>Average +/- exp +/- (mb, theory)</td>
<td>4.49</td>
<td>± 0.19</td>
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</tr>
</tbody>
</table>

$\chi^2$/dof = 6.1/6 (CL = 40.7%)

**OPE-HQET-SCET (BLNP)**


$m_b$ input from $b \rightarrow c l \nu$ and $b \rightarrow s \gamma$ moments

Average +/- exp +/- (mb, theory)

4.49 ± 0.19 ± 0.27

$\chi^2$/dof = 6.1/6 (CL = 40.7%)

**Dressed Gluon Exponentiation (DGE)**

JHEP 0601:097,2006

$m_b$ input from $b \rightarrow c l \nu$ and $b \rightarrow s \gamma$ moments

4.49 ± 0.19 ± 0.27

$\chi^2$/dof = 6.1/6 (CL = 40.7%)
Non-leptonic decays require the calculation of hadronic matrix elements of four-quark operators, e.g. for a decay like $B \to \pi\pi$

$$\mathcal{M} = \langle B| (\bar{b}\gamma_\mu(1 - \gamma_5)q)(\bar{q}'\gamma_\mu(1 - \gamma_5)q'')|\pi\pi\rangle$$

In the large $m_b$ limit a factorization theorem has been proven (QCD-Factorization, similar to SCET)

(Beneke, Buchalla, Neubert, Sachrajda)
General Properties of QCD-Factorization

- Leading term is (contains) naive factorization
- Non-perturbative quantities are the soft form factor and the light cone distributions of the light hadrons and of the $B$ meson
- The strong phases of the matrix elements are either perturbative ($\mathcal{O}(\alpha_s(m_b))$) or power suppressed ($\mathcal{O}(\Lambda_{QCD}/m_b)$)
- → The strong phases are predicted to be small(ish)
- → Important for the calculation of CP Asymmetries
- Does it work?
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Does it work?
Introduction / Motivation
Ancient Wisdom
Inclusive Decays
Recent Developments

Soft Collinear Effective Theory
Towards Understanding Nonleptonic Decays

\( \frac{2B(\pi^0 K^\pm)}{B(\pi^\pm K^0)} \)
\( \frac{B(\pi^+ K^\pm)}{2B(\pi^0 K^0)} \)
\( \frac{B(\pi^+ \pi^-)}{B(\pi^\mp K^\pm)} \)

\( \frac{\tau_{B^+}^{\pm}}{\tau_{B^0}^0} \frac{B(\pi^+ K^\pm)}{B(\pi^\pm K^0)} \)
\( \frac{\tau_{B^+}^{\pm}}{\tau_{B^0}^0} \frac{B(\pi^+ \pi^-)}{2B(\pi^\pm \pi^0)} \)
\( \frac{\tau_{B^+}^{\pm}}{\tau_{B^0}^0} \frac{2B(K^0 \pi^0)}{B(K^\pm \pi^0)} \)


Thomas Mannel, University of Siegen
Theoretical Tools for Heavy Quark Physics
Update on the BR's by Neubert (CKM 2005, San Diego)
Update on the CP Asymmetries by Neubert (CKM 2005)
Effective field theory methods made precise calculations in heavy quark physics possible. Starting about 1989 HQET and HQE put heavy quark physics on a model independent basis. Model dependence often appears only at subleading orders. SCET is an ansatz to understand also exclusive non-leptonic decays systematically. I did not talk about Lattice QCD calculations: Enormous progress due to better algorithms and to stronger computers. Heavy Flavour Physics has become (in some corners) a precision field.
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1988: Pre-historic Unitarity Triangle
2006: Today's Unitarity Triangle

![Diagram showing the unitarity triangle with exclusion areas and parameters such as $\rho$, $\eta$, $\sin 2\beta$, $\epsilon_K$, $|V_{ub}/V_{cb}|$, and solutions with $\cos 2\beta < 0$ excluded at CL > 0.95.]

BEAUTY 2006

Thomas Mannel, University of Siegen

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