Theory Overview
of Semileptonic Decays

Thomas Mannel
Theoretical Particle Physics, University of Siegen

April 7th, 2011
BEAUTY 2011, Amsterdam
Contents

1 Theory Tools
   - Heavy Quark Expansion
   - Modified Heavy Quark Expansion
   - Exclusive Decays: $b \rightarrow c$
   - Exclusive Decays: $b \rightarrow u$: Updated LCQCDSR Result

2 Theoretical Issues
   - Input Parameters
   - The Role of $B \rightarrow \tau \bar{\nu}$

3 Conclusions and Outlook
Introduction: Why is this of interest?

- **Important Ingredient for the Unitarity Traingle**

- **Standard Fit of for the Unitarity Traingle**

- **“Unitarity Clock”:**
  \[ \left| V_{ub} / V_{cb} \right| \]

- **Relation between Kaon CP violation and the Unitarity Triangle**

- Thomas Mannel, Uni. Siegen

Semileptonic Theory
Theoretical Tools

- There is a large toolbox:
  - For inclusive decays:
    - Heavy Quark Expansion: Local OPE
    - Heavy Quark Expansion: Shape functions and Soft Collinear Effective Theory (SCET)
  - For exclusive decays:
    - Heavy Quark Effective Theory (HQET)
    - QCD (Light Cone) Sum Rules
    - Lattice
- Models are completely outdated!
- Precision Methods with controllable uncertainties
Heavy Quark Expansion

Heavy Quark Expansion = Operator Product Expansion
(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, M,...)

\[
\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) \left| \langle X | \mathcal{H}_{\text{eff}} | B(\nu) \rangle \right|^2
\]

\[
= \int d^4x \langle B(\nu) | \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) | B(\nu) \rangle
\]

\[
= 2 \text{Im} \int d^4x \langle B(\nu) | T \{ \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) \} | B(\nu) \rangle
\]

\[
= 2 \text{Im} \int d^4x \ e^{-im_b \nu \cdot x} \langle B(\nu) | T \{ \tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}_{\text{eff}}(0) \} | B(\nu) \rangle
\]

Last step: \( b(x) = b_v(x) \exp(-im_{\nu}vx) \),
corresponding to \( p_b = m_b \nu + k \)

Expansion in the residual momentum \( k \)
- Perform an “OPE”: $m_b$ is much larger than any scale appearing in the matrix element

$$
\int d^4 x e^{-i m_b v x} T\{ \tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}_{\text{eff}}^\dagger(0) \} = \sum_{n=0}^{\infty} \left( \frac{1}{2m_Q} \right)^n C_{n+3}(\mu) O_{n+3}
$$

→ The rate for $B \rightarrow X_c \ell \bar{\nu}_\ell$ can be written as

$$
\Gamma = \Gamma_0 + \frac{1}{m_Q} \Gamma_1 + \frac{1}{m_Q^2} \Gamma_2 + \frac{1}{m_Q^3} \Gamma_3 + \cdots
$$

- The $\Gamma_i$ are power series in $\alpha_s(m_Q)$:
  → Perturbation theory!

- Works also for differential rates!
\( \Gamma_0 \) is the decay of a free quark ("Parton Model")
\( \Gamma_1 \) vanishes due to Heavy Quark Symmetries
\( \Gamma_2 \) is expressed in terms of two parameters

\[
2M_H \mu_\pi^2 = - \langle H(v) | \bar{Q}_v (iD)^2 Q_v | H(v) \rangle \\
2M_H \mu_G^2 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu)(iD^\nu) Q_v | H(v) \rangle
\]

\( \mu_\pi \): Kinetic energy and \( \mu_G \): Chromomagnetic moment

\( \Gamma_3 \) two more parameters

\[
2M_H \rho_D^3 = - \langle H(v) | \bar{Q}_v (iD_\mu)(ivD)(iD^\mu) Q_v | H(v) \rangle \\
2M_H \rho_{LS}^3 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu)(ivD)(iD^\nu) Q_v | H(v) \rangle
\]

\( \rho_D \): Darwin Term and \( \rho_{LS} \): Spin-Orbit Term

\( \Gamma_4 \) and \( \Gamma_5 \) have been computed Bigi, Uraltsev, Turczyk, TM, ...

Thomas Mannel, Uni. Siegen | Semileptonic Theory
Structure of the HQE

Structure of the expansion (@ tree):

\[ d\Gamma = d\Gamma_0 + \left( \frac{\Lambda_{QCD}}{m_b} \right)^2 d\Gamma_2 + \left( \frac{\Lambda_{QCD}}{m_b} \right)^3 d\Gamma_3 + \left( \frac{\Lambda_{QCD}}{m_b} \right)^4 d\Gamma_4 \]

\[ + d\Gamma_5 \left( a_0 \left( \frac{\Lambda_{QCD}}{m_b} \right)^5 + a_2 \left( \frac{\Lambda_{QCD}}{m_b} \right)^3 \left( \frac{\Lambda_{QCD}}{m_c} \right)^2 \right) \]

\[ + \ldots + d\Gamma_7 \left( \frac{\Lambda_{QCD}}{m_b} \right)^3 \left( \frac{\Lambda_{QCD}}{m_c} \right)^4 \]

- \( d\Gamma_3 \propto \ln \left( \frac{m_c^2}{m_b^2} \right) \)
- Power counting \( m_c^2 \sim \Lambda_{QCD} m_b \)
Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
  Bigi, Zwicky, Uraltsev, Turczyk, TM, ...

- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
  Melnikov, Czarnecki, Pak

- $\mathcal{O}(\alpha_s)$ for the $\mu^2/\mu^2 \to m_b^2$ is known
  Becher, Boos, Lunghi, Gambino

- In the pipeline:
  - Complete $\alpha_s/m_b^2$, including the $\mu_G$ terms
  - More on the “Intrinsic charm” and “weak annihilation” contributions

A theo. uncertainty of 1% in $V_{cb,incl}$ looks plausible!
Problem: Cuts needed to suppress charmed decays

Forces us into corners of phase space, where the usual OPE breaks down

Expansion parameter $\Lambda_{\text{QCD}}/(m_b - 2E_\ell)$

Instead of HQE Parameters: Shape Functions $f(\omega)$

$$2M_B f(\omega) = \langle B(v) | \bar{b}_v \delta(\omega + i(n \cdot D)) | B(v) \rangle$$

Universal for all heavy-to-light decays

Systematics: SoftCollinearEffectiveTheory calculation

Several subleading shape functions

perturbative QCD corrections
Shape Functions

- Shape function vs. local OPE: Moment Expansion

\[ f(\omega) = \delta(\omega) + \frac{\mu_{\pi}^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots \]

- Perturbative “jetlike” contributions: Convolution

\[ S(\omega, \mu) = \int dk \ C_0(\omega - k, \mu)f(k) \]

- Charged Lepton Energy Spectrum \( (H: \text{hard QCD corrections}) \)

\[ \frac{d\Gamma}{dy} = \frac{G_F^2|V_{ub}^2|m_b^5}{96\pi^3} \int d\omega \Theta(m_b(1 - y) - \omega)H(\mu)S(\omega, \mu) \]
Approaches

- Obtaining the Shape functions:
  - From Comparison with $B \rightarrow X_s\gamma$
  - From the knowledge of (a few) moments
  - From modeling

- QCD based:
  - BLNP (Bosch, Lange, Neubert, Paz)
  - GGOU (Gambino, Giordano, Ossola, Uraltsev)
  - SIMBA (Tackmann, Tackmann, Lacker, Liegti, Stewart ...)

- QCD inspired:
  - Dressed Gluon Exponentiation (Andersen, Gardi)
  - Analytic Coupling (Aglietti et al.)

- Attempts to avoid the shape functions (Bauer Ligeti, Luke ...)

Theo. uncertainty in $V_{ub, incl}$ is still (7 ... 10) %
Exclusive \( b \to c \) Decays

- Kinematic variable for a heavy quark: Four Velocity \( \nu \)
- Differential Rates

\[
\frac{d\Gamma}{d\omega}(B \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_D^3 (\omega^2 - 1)^{1/2} P(\omega) (\mathcal{F}(\omega))^2
\]

\[
\frac{d\Gamma}{d\omega}(B \to D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2
\]

- with \( \omega = \nu \nu' \) and
- \( P(\omega) \): Calculable Phase space factor
- \( \mathcal{F} \) and \( \mathcal{G} \): Form Factors
Heavy Quark Symmetries

- Normalization of the Form Factors is known at $\nu \nu' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\text{QED}} \eta_A \left[ 1 + \frac{\delta_1}{\mu^2} + \cdots \right] + (\omega - 1) \rho^2 + \mathcal{O}((\omega - 1)^2)$$

$$\mathcal{G}(1) = \eta_{\text{QED}} \eta_V \left[ 1 + \mathcal{O} \left( \frac{m_B - m_D}{m_B + m_D} \right) \right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_1/\mu^2 = -(8 \pm 4)\%$, $\eta_{\text{QED}} = 1.007$
Unquenched Calculations become available!

Heavy Mass Limit is not used

Lattice Calculations of the deviation from unity

\[ F(1) = 0.908 \pm 0.016 \]

\[ G(1) = 1.074 \pm 0.018 \pm 0.016 \]

**B → D(∗) Form Factors: Non-Lattice Results**

- **B → D* Form Factor:**
  - Based on Zero Recoil Sum Rules (Uraltsev, also Ligeti et al.)
  - Including full $\alpha_s$ and up to $1/m_b^5$

  $\mathcal{F}(1) = 0.86 \pm 0.04$  
  (Gambino, Uraltsev, M (2010))

- **B → D Form Factor:**
  - Based on the “BPS limit” $\mu_{\pi}^2 = \mu_G^2$

  $\mathcal{G}(1) = 1.04 \pm 0.02$  
  (Uraltsev)

The tension between $V_{cb,\text{incl}}$ and $V_{cb,\text{excl}}$ is about to disappear!
Exclusive Decays: \( b \rightarrow u \)

- Focus on \( B \rightarrow \pi \ell \bar{\nu}_\ell \)
- Hadronic Matrix Element:
  \[
  \langle \pi(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = f^+_B(q^2)(2p+q)_\mu + \text{Terms with } f^0_{B\pi}(q^2)
  \]
- Differential rate:
  \[
  \frac{d\Gamma(\bar{B}^0 \rightarrow \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3} p^3 \pi f^+_B(q^2)^2 + O(m_l^2)
  \]
- Measurements are quickly improving
- Shape constrained by analyticity
- Need the normalization \( f^+_B(0) \)
- In case \( \ell = \tau \) also \( f_0(q^2) \) s needed!
Tools: Form Factor Parametrizations

- **Becirevic Kaidalov Parametrization**
  
  \[ f_+ (q^2) = \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)} \]

- **z parametrization** (Arnesen et al., Boyd, Grinstein, Lebed)
  
  \[ P(t)\phi(t, t_0)f_+(t) = \sum_{k=0}^{\infty} a_k(t_0) z^k(t, t_0) \]

  with

  \[ z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_B + m_\pi)^2 \]
Status of LCSR calculation


\[ \Delta \zeta (0, q_{max}^2) = \frac{1}{|V_{ub}|^2 \tau_{B^0}} \int_{0}^{q_{max}^2} dq^2 \frac{d\beta(B \to \pi \ell \nu_{\ell})}{dq^2}, \]

- including
  - Full $O(\alpha_s)$ QCD corrections
  - Subleading twists
  - $a_2$ and $a_4$ corrections to the pion DA, fitted from the electromagnetic pion form factor
The pion e.m. form factor calculated from LCSR [16, 17] as a function of Gegenbauer moments $a_2^\pi(1 \text{ GeV})$ and $a_4^\pi(1 \text{ GeV})$ and fitted (solid) to the experimental data points taken from from [18].

$$a_2^\pi(1 \text{ GeV}) = 0.17 \pm 0.08, \quad a_4^\pi(1 \text{ GeV}) = 0.06 \pm 0.10$$
LCQCDSR Result for the form factor, $0 \leq q^2 \leq 12 \text{ GeV}^2$

(a): $a_2^\pi$, $a_4^\pi$, (b): $\mu_\pi$, (c): $\mu$, (d): $M^2$, $s_0^B$
Linking high $q^2$ with low $q^2$

- LCQCDSR are limited to “small” values of $q^2$
- Complementary to lattice calculations
- We have QCD based calculations / estimates of the from factors $f_+$ and $f_0$ in the full kinematic region
- Uncertainties become controllable and are already quite small!
- May become the most accurate way to determine $V_{ub}$
Linking high $q^2$ with low $q^2$: $z$ parametrization

The vector form factor $f_{B\pi}^+(q^2)$ calculated from LCSR and fitted to the BCL parameterization (solid) with uncertainties (dashed), compared with the HPQCD [4] (triangles) and FNAL/MILC [5] (squares) results.
Theory vs. Experiment

(colour online) The normalized $q^2$-distribution in $B \rightarrow \pi l \nu$ obtained from LCSR and extrapolated with the z-series parameterization (central input- solid, uncertainties -dashed). The experimental data points are from BABAR: (red) squares [1], (blue) triangles [2] and Belle [3]: (magenta) full circles.
Value of $V_{ub}$ from this work:

$$|V_{ub}| = (3.50^{+0.38}_{-0.33}|_{th.} \pm 0.11|_{exp.}) \times 10^{-3}$$

Lattice $\otimes$ LCQCDSR has reached 10% th. uncertainty in $V_{ub, excl}$. 
Inputs for the standard OPE

- Two, in principle equivalent schemes: kinetic scheme and 1S scheme

<table>
<thead>
<tr>
<th>O(1)</th>
<th>Kinetic scheme</th>
<th>1S scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_b, m_c</td>
<td></td>
<td>m_b</td>
</tr>
<tr>
<td>O(1/m_b^2)</td>
<td>μ^2_π, μ^2_G</td>
<td>λ_1, λ_2</td>
</tr>
<tr>
<td>O(1/m_b^3)</td>
<td>ρ_D, ρ_LS</td>
<td>ρ_1, τ_1-3</td>
</tr>
</tbody>
</table>

- Parameters are determined from the spectra: hadronic invariant mass, charged lepton and hadronic energy
The semileptonic moments identify only a strip $m_b - 0.6m_c$ in the $m_b, m_c$ plane!
Recent Mass Determinations

(Plot from Paolo Gambino)

Recent sum rules determinations converted to kin scheme

PDG07

Boughezal et al 07

Hoang, Jamin 04

Kuehn et al 07

(Plot from Paolo Gambino)
HQE Parameters

- These may be determined from the spectra
  - Hadronic invariant mass
  - Charged Lepton Energy
  - Hadronic Energy
- Moments in terms of a $1/m_b$ expansion
- Higher Moments $\Leftrightarrow$ Higher Dimensional Operators
- Lepton-Energy Cut dependence of the moments can be reliably calculated
- Determination of $m_b$, $m_c$, $\mu_\pi^2$, $\mu_G^2$, ... from data
Reasonably good determination of the mass and the HQE parameters (up to $1/m_b^3$)
Shape Functions

- Shape functions can be extracted from $B \rightarrow X_s \gamma$
- Several sub-leading shape functions!
- Attempt for a systematic fit: SIMBA
  (Tackmann, Tackmann, Lacker, Liegti, Stewart ...)
  - Systematic expansion in terms of basis functions
    \[
    F(\lambda x) = \frac{1}{\lambda} \left[ \sum_{n=0}^{\infty} c_n f_n(x) \right]^2 
    \int dk \ F(k) = 1 = \sum_{n=0}^{\infty} c_n^2
    \]
- Reduce the cut dependences
Chose different bases:
Check for basis independence
The Role of $B \rightarrow \tau \bar{\nu}$

- $B \rightarrow \tau \bar{\nu}$ depends crucially on $f_B$

  $$\mathcal{B}(B^- \rightarrow \tau \bar{\nu}) = \frac{G_F^2}{8\pi} |V_{ub}|^2 m^2 m_B \left(1 - \frac{m^2}{m_B^2}\right)^2 f^2_{B \tau B^-}$$

- The extracted $V_{ub}$ value is quite large ...
- However, if the data are right, QCD (or the SM) must have a problem: Define

  $$R_{s/l}(q_1^2, q_2^2) \equiv \frac{\Delta \mathcal{B}_{B \rightarrow \pi \ell \nu_\ell}(q_1^2, q_2^2)}{\mathcal{B}(B \rightarrow \tau \nu_\tau)} \left(\frac{\tau_{B^-}}{\tau_{B^0}}\right)$$

  $$= \frac{\Delta \zeta(q_1^2, q_2^2)}{(G_F^2/8\pi)m^2 m_B(1 - m^2/m_B^2)^2 f^2_B}$$

Thomas Mannel, Uni. Siegen
### Input Parameters

The Role of $B \rightarrow \tau \bar{\nu}$

<table>
<thead>
<tr>
<th>Exp.</th>
<th>$\Delta B(10^{-4})$ [Ref.]</th>
<th>$\mathcal{B}(B \rightarrow \tau \nu_\tau)(10^{-4})$ [Ref.]</th>
<th>$R_{s/l}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BABAR</td>
<td>$0.32 \pm 0.03$ [1]</td>
<td>$1.76 \pm 0.49$ [36, 37]</td>
<td>$0.20^{+0.08}_{-0.05}$</td>
</tr>
<tr>
<td></td>
<td>$0.33 \pm 0.03 \pm 0.03$ [2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belle</td>
<td>$0.398 \pm 0.03$ [3]</td>
<td>$1.54^{+0.38+0.29}_{-0.37-0.31}$ [38]</td>
<td>$0.28^{+0.13}_{-0.07}$</td>
</tr>
<tr>
<td>QCD</td>
<td>$\Delta \zeta (ps^{-1})$ [Ref.]</td>
<td>$f_B$(MeV) [Ref.]</td>
<td>$R_{s/l}$</td>
</tr>
<tr>
<td>HPQCD</td>
<td>$2.02 \pm 0.55$ [4]</td>
<td>$190 \pm 13$ [34]</td>
<td>$0.52 \pm 0.16$</td>
</tr>
<tr>
<td>FNAL/MILC</td>
<td>$2.21^{+0.57}_{-0.42}$ [5]</td>
<td>$212 \pm 9$ [35]</td>
<td>$0.46 \pm 0.10$</td>
</tr>
</tbody>
</table>

$R_{s/l}$ for the region $16 \text{ GeV}^2 < q^2 < 26.4 \text{ GeV}^2$
### Table: \( R_{s/l} \) for the region \( 0 \text{ GeV}^2 < q^2 < 12.0 \text{ GeV}^2 \)

<table>
<thead>
<tr>
<th>Exp.</th>
<th>( \Delta B(10^{-4}) ) [Ref.]</th>
<th>( \mathcal{B}(B \to \tau \nu_{\tau})(10^{-4}) ) [Ref.]</th>
<th>( R_{s/l} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BABAR</td>
<td>0.88 ± 0.06 [1]</td>
<td>1.76 ± 0.49 [36, 37]</td>
<td>0.52^{+0.20}_{-0.12}</td>
</tr>
<tr>
<td></td>
<td>0.84 ± 0.03 ± 0.04 [2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>QCD</td>
<td>( \Delta \zeta ) [Ref.]</td>
<td>( f_B) (MeV) [Ref.]</td>
<td>( R_{s/l} )</td>
</tr>
<tr>
<td>LCSR/QCDSR</td>
<td>4.59^{+1.00}_{-0.85} [this work]</td>
<td>210 ± 19 [41]</td>
<td>0.97^{+0.28}_{-0.24}</td>
</tr>
</tbody>
</table>

Some clarification is needed here ...
Conclusions and Outlook

- $V_{cb}$ is in good shape
  - OPE in inclusive method very well understood
  - Excl. vs. Incl. tension is becoming smaller
  - Calculation of $\alpha_s \mu^2_G$ terms

- The tension in inc. vs. excl. $V_{ub}$ stays with us ...
  - Form factors are pretty well constrained
  - Smaller uncertainties in $V_{ub}$ from $B \to \pi \ell \bar{\nu}$
  - Scrutinize both inclusive and exclusive methods
  - It is not yet time to speculate about new physics in $b \to u$ semileptonics ...

- What is going on in $B \to \tau \bar{\nu}$?