Part I: Heavy Quark Effective Theory

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KITPC, June 24th, 2008
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Introduction and Motivation

What is interesting about heavy quarks?

- Weak decays: Access to fundamental parameters
  - CKM Matrix Elements
  - CP violating parameters
  - new physics ??

- Heavy mass makes QCD controllable:
  - Heavy Quark Effective Theory (Exclusive Decays)
  - Heavy Quark Expansion (Inclusive Decays)
Flavour Physics (of quarks):
Transitions between different kinds of Quarks

Its all about weak interactions ...
Strong interactions as a “background”
Motivation
Effective Field Theories
Heavy mass limit

Weak Interaction Reminder
Peculiarties of the Standard Model

Quark Mixing in the Standard Model

- Mass term for the Up Quarks
  \[ \mathcal{L}^u_{\text{mass}} = -\bar{U}_L \mathcal{M}^u U_R + \text{h.c.} \]

- Mass term for the Down Quarks
  \[ \mathcal{L}^d_{\text{mass}} = -\bar{D}_L \mathcal{M}^d D_R + \text{h.c.} \]

- 3 × 3 Mass Matrices \( \mathcal{M}^u \) and \( \mathcal{M}^d \): \[ [\mathcal{M}^u, \mathcal{M}^d] \neq 0 \]

- relative Rotation between the two Eigenbases:
  \[ \mathcal{M}^u = \mathcal{M}^u_{\text{diag}} \quad \text{and} \quad \mathcal{M}^d = V_{\text{CKM}} \mathcal{M}^d_{\text{diag}} V_{\text{CKM}}^\dagger \]

- \( V_{\text{CKM}} \): CKM Matrix
Motivation  
Effective Field Theories  
Heavy mass limit  
Weak Interaction Reminder  
Peculiarities of the Standard Model

Charged and Neutral Currents

\[ V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

- CKM Matrix appears only in the charged currents:
  \[ L_{cc-int} = \frac{g}{\sqrt{2}} \bar{U}_L (\gamma \cdot W^\pm) V_{CKM} D_L \]

- Neutral currents remain flavour diagonal: (at tree level)
  \[ L_{nc-int} = \frac{g}{\sqrt{2}} \bar{U}_L (\Gamma \cdot Z^0) 1 U_L + \cdots \]

- CP Violation through the irreducible phases of CKM

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Peculiarities of SM Flavour Mixing

- Hierarchical structure of the CKM matrix
- Quark Mass spectrum is widely spread
  \( m_u \sim 10 \text{ MeV} \) to \( m_t \sim 170 \text{ GeV} \)
- PMNS Matrix for lepton flavour mixing is not hierarchical
- Only the charged lepton masses are hierarchical
  \( m_e \sim 0.5 \text{ MeV} \) to \( m_\tau \sim 1772 \text{ MeV} \)
- Up-type leptons \( \sim \) Neutrinos have very small masses
- (Enormous) Suppression of Flavour Changing Neutral Currents:
  \( b \rightarrow s, \ c \rightarrow u, \ \tau \rightarrow \mu, \ \mu \rightarrow e, \ \nu_2 \rightarrow \nu_1 \)
Peculiarities of SM CP Violation

- Strong CP remains mysterious
- Flavour diagonal CP Violation is well hidden:
  e.g. electric dipole moment of the neutron:
  At least three loops (Shabalin)

\[
d_e \sim e \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \text{Im}\Delta \mu^3
\]
\[
\sim 10^{-32} \text{ e cm} \quad \text{with} \quad \mu \sim 0.3 \text{ GeV}
\]
\[
d_{\exp} \leq 3.0 \times 10^{-26} \text{ e cm}
\]
Effective Field Theories

- Weak decays:
  - Very different mass scales are involved:
    - $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$: Scale of strong interactions
    - $m_c \sim 1.5 \text{ GeV}$: Charm Quark Mass
    - $m_b \sim 4.5 \text{ GeV}$: Bottom Quark Mass
    - $m_t \sim 175 \text{ GeV}$ and $M_W \sim 81 \text{ GeV}$: Top Quark Mass and Weak Boson Mass
    - $\Lambda_{\text{NP}}$ Scale of “new physics”
  - At low scales the high mass particles / high energy degrees of freedom are irrelevant.
  - Construct an “effective field theory” where the massive / energetic degrees of freedom are removed (“integrated out”)
Integrating out heavy degrees of freedom

- $\phi$: light fields, $\Phi$: heavy fields with mass $\Lambda$
- Generating functional as a functional integral

\[ Z[j] = \int [d\phi][d\Phi] \exp \left( \int d^4x [L(\phi, \Phi) + j\phi] \right) \]

\[ = \int [d\phi] \exp \left( \int d^4x [L_{\text{eff}}(\phi) + j\phi] \right) \quad \text{with} \]

\[ \exp \left( \int d^4x L_{\text{eff}}(\phi) \right) = \int [d\Phi] \exp \left( \int d^4x L(\phi, \Phi) \right) \]
For length scales $x \gg 1/\Lambda$: local effective Lagrangian

Technically: *(Operator Product) Expansion* in inverse powers of $\Lambda$

$$\mathcal{L}_{\text{eff}}(\phi) = \mathcal{L}_{\text{eff}}^{(4)}(\phi) + \frac{1}{\Lambda} \mathcal{L}_{\text{eff}}^{(5)}(\phi) + \frac{1}{\Lambda^2} \mathcal{L}_{\text{eff}}^{(6)}(\phi) + \cdots$$

$\mathcal{L}_{\text{eff}}$ is in general non-renormalizable, but ...

$\mathcal{L}_{\text{eff}}^{(4)}$ is the renormalizable piece

For a fixed order in $1/\Lambda$: Only a finite number of insertions of $\mathcal{L}_{\text{eff}}^{(4)}$ is needed!

$\rightarrow$ can be renormalized

Renormalizability is not an issue here
Motivation
Effective Field Theories
Heavy mass limit

EFT in a nutshell
Example: Weak Hamiltonian
Introduction to Renormalization Group

\[ \mu = M_W, m_t \]  Weak Gauge Bosons. Top Quark

\[ \mu = m_b \]  Mass of the b Quark

\[ \mu = m_c \]  Mass of the charm quark

\[ \mu = \Lambda_{QCD} \]  Hadronic Scale

Renormalization
Renormalization
Renormalization

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Example: Weak Hamiltonian

- Start out from the Standard Model
- $W^\pm, Z^0, \text{top:}$ much heavier than any hadron mass
- “integrate out” these particles at the scale $\mu \sim M_{\text{Hadron}}$

$W$ has zero range in this limit:

$$\langle 0 | T[W^*_\mu(x) W_\nu(y)] | 0 \rangle \to g_{\mu\nu} \frac{1}{M_W^2} \delta^4(x - y)$$

Effective Interaction (Fermi Coupling)

$$H_{\text{eff}} = \frac{g^2}{\sqrt{2} M_W^2} V_{q'q} [\bar{q}' \gamma_\mu (1 - \gamma_5) q] [\bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \nu_\ell] = \frac{4G_F}{\sqrt{2}} V_{q'q} j_{\mu, \text{had}} j^{\mu}_{\text{lep}}$$
Renormalization Group Running

- $H_{\text{eff}}$ is defined at the scale $\Lambda$, where we integrated out the particles with mass $\Lambda$: **General Structure**

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}}\lambda_{\text{CKM}} \sum_k \hat{C}_k(\Lambda)O_k(\Lambda)$$

- $O_k(\Lambda)$: The matrix elements of $O_k$ have to be evaluated ("normalized") at the scale $\Lambda$.
- $\hat{C}_k(\Lambda)$: Short distance contribution, contains the information about scales $\mu > \Lambda$
- Matrix elements of $O_k(\Lambda)$: Long Distance Contribution, contains the information about scales $\mu < \Lambda$
We could as well imagine a situation with a different definition of “long” and “short” distances, defined by a scale $\mu$, in which case

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k C_k(\Lambda/\mu) \mathcal{O}_k(\mu)$$

Key Observation: The matrix elements of $H_{\text{eff}}$ are physical Quantities, thus cannot depend on the arbitrary choice of $\mu$

$$0 = \mu \frac{d}{d\mu} H_{\text{eff}}$$

compute this ....
\[ 0 = \sum_i \left( \mu \frac{d}{d\mu} C_i(\Lambda/\mu) \right) O_i(\mu) + C_i(\Lambda/\mu) \left( \mu \frac{d}{d\mu} O_i(\mu) \right) \]

**Operator Mixing:** Change in scale can turn the operator \( O_i \) into a linear combination of operators (of the same dimension)

\[ \mu \frac{d}{d\mu} O_i(\mu) = \sum_j \gamma_{ij}(\mu) O_j(\mu) \]

and so

\[ \sum_i \sum_j \left( \left[ \delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}(\mu) \right] C_i(\Lambda/\mu) \right) O_j(\mu) = 0 \]
Assume: The operators $O_j$ from a basis, then

$$\sum_i \left[ \delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}^T(\mu) \right] C_j(\Lambda/\mu) = 0$$

QCD: Coupling constant $\alpha_s$ depends on $\mu$: $\beta$-function

$$\mu \frac{d}{d\mu} \alpha_s(\mu) = \beta(\alpha_s(\mu))$$

$C_j$ depend also on $\alpha_s$

$$\mu \frac{d}{d\mu} = \left( \mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right)$$

In an appropriate scheme $\gamma_{ij}$ depend on $\mu$ only trough $\alpha_s$:

$$\gamma_{ij}(\mu) = \gamma_{ij}(\alpha_s(\mu))$$
Renormalization Group Equation (RGE) for the coefficients

\[ \sum_i \left[ \delta_{ij} \left( \mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) + \gamma_{ij}^T (\alpha_s) \right] C_j(\Lambda/\mu, \alpha_s) = 0 \]

This is a system of linear differential equations:

→ Once the initial conditions are known, the solution is in general unique

RGE Running: Use the RGE to relate the coefficients at different scales
The coefficients are at $\mu = \Lambda$ (at the “matching scale”)

$$C_i(\Lambda/\mu = 1, \alpha_s) = \sum_n a_i^{(n)} \left( \frac{\alpha_s}{4\pi} \right)^n$$

perturbative calculation

Perturbative calculation of the RG functions $\beta$ and $\gamma_{ij}$

$$\beta(\alpha_s) = \alpha_s \sum_{n=0} \beta^{(n)} \left( \frac{\alpha_s}{4\pi} \right)^{n+1}$$

$$\gamma_{ij}(\alpha_s) = \sum_{n=0} \gamma_{ij}^{(n)} \left( \frac{\alpha_s}{4\pi} \right)^{n+1}$$

RG functions can be calculated from loop diagrams:

$$\beta^{(0)} = -\frac{2}{3} (33 - 2n_f)$$

$\gamma_{ij}$ depends on the set of $\mathcal{O}_i$
Structure of the perturbative expansion of the coefficient at some other scale

\[ c_i(\Lambda/\mu, \alpha_s) = \]

\[ b_i^{00} + b_i^{11} \left( \frac{\alpha_s}{4\pi} \right) \ln \frac{\Lambda}{\mu} + b_i^{10} \left( \frac{\alpha_s}{4\pi} \right) \]

\[ + b_i^{22} \left( \frac{\alpha_s}{4\pi} \right)^2 \ln^2 \frac{\Lambda}{\mu} + b_i^{21} \left( \frac{\alpha_s}{4\pi} \right)^2 \ln \frac{\Lambda}{\mu} + b_i^{20} \left( \frac{\alpha_s}{4\pi} \right)^2 \]

\[ + b_i^{33} \left( \frac{\alpha_s}{4\pi} \right)^3 \ln^3 \frac{\Lambda}{\mu} + b_i^{32} \left( \frac{\alpha_s}{4\pi} \right)^3 \ln^2 \frac{\Lambda}{\mu} + b_i^{31} \left( \frac{\alpha_s}{4\pi} \right)^3 \ln \frac{\Lambda}{\mu} + \cdots , \]
LLA (Leading Log Approximation): Resummation of the $b_i^{nn}$ terms

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} b_i^{nn} \left( \frac{\alpha_s}{4\pi} \right)^n \ln^n \frac{\Lambda}{\mu}$$

→ leading terms in the expansion of the RG functions

NLLA (Next-to-leading log approximation):

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} \left[ b_i^{nn} + b_i^{n+1,n} \left( \frac{\alpha_s}{4\pi} \right) \right] \left( \frac{\alpha_s}{4\pi} \right)^n \ln^n \frac{\Lambda}{\mu}$$

→ next-to-leading terms of the RG functions
Typical Procedure:

- “Matching” at the scale $\mu = M_W$
- “Running” to a scale of the order $\mu = m_b$
- $\rightarrow$ includes operator mixing

Resummation of the large logs $\ln(M_W^2/m_b^2)$

- “Matching” at the scale $\mu = m_b$
- “Running” to the scale $m_c$

Resummation of the “large” logs $\ln(m_b^2/m_c^2)$

... until $\alpha_s(\mu)$ becomes too large ...
Heavy Quark Limit

- 1/$m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b, m_c \to \infty$ with fixed (four)velocity
  \[ v_Q = \frac{p_Q}{m_Q}, \quad Q = b, c \]
- In this limit we have
  \[ \begin{align*}
  m_{\text{Hadron}} &= m_Q \\
  p_{\text{Hadron}} &= p_Q \\
  v_{\text{Hadron}} &= v_Q
  \end{align*} \]
- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics!
**Heavy Quark Symmetries**

- The interaction of gluons is **identical for all quarks**
- Flavour enters QCD only through the mass terms
  - $m \rightarrow 0$: (Chiral) Flavour Symmetry (Isospin)
  - $m \rightarrow \infty$ Heavy Flavour Symmetry
- Consider $b$ and $c$ heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons:

$$H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B})Q \quad m_Q \rightarrow \infty \rightarrow 0$$

- Spin Rotations become a symmetry
- Heavy Quark Spin Symmetry: SU(2) Rotations
- Spin Flavour Symmetry Multiplets
Mesonic Ground States

Bottom:

\[ |(b\bar{u})_{J=0}\rangle = |B^-\rangle \]
\[ |(b\bar{d})_{J=0}\rangle = |\bar{B}^0\rangle \]
\[ |(b\bar{s})_{J=0}\rangle = |\bar{B}_s\rangle \]
\[ |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle \]
\[ |(b\bar{d})_{J=1}\rangle = |\bar{B}^{*0}\rangle \]
\[ |(b\bar{s})_{J=1}\rangle = |\bar{B}_s^*\rangle \]

Charm:

\[ |(c\bar{u})_{J=0}\rangle = |D^0\rangle \]
\[ |(c\bar{d})_{J=0}\rangle = |D^+\rangle \]
\[ |(c\bar{s})_{J=0}\rangle = |D_s\rangle \]
\[ |(c\bar{u})_{J=1}\rangle = |D^{*0}\rangle \]
\[ |(c\bar{d})_{J=1}\rangle = |D^{*+}\rangle \]
\[ |(c\bar{s})_{J=1}\rangle = |D_s^*\rangle \]
Mesonic Ground States

**Bottom:**

\[
\begin{align*}
| (b \bar{u})_{J=0} \rangle &= | B^- \rangle \\
| (b \bar{d})_{J=0} \rangle &= | B^0 \rangle \\
| (b \bar{s})_{J=0} \rangle &= | B_s \rangle \\
| (b \bar{u})_{J=1} \rangle &= | B^{*-} \rangle \\
| (b \bar{d})_{J=1} \rangle &= | B^{*0} \rangle \\
| (b \bar{s})_{J=1} \rangle &= | B_s^{*} \rangle
\end{align*}
\]

**Charm:**

\[
\begin{align*}
| (c \bar{u})_{J=0} \rangle &= | D^0 \rangle \\
| (c \bar{d})_{J=0} \rangle &= | D^+ \rangle \\
| (c \bar{s})_{J=0} \rangle &= | D_s \rangle \\
| (c \bar{u})_{J=1} \rangle &= | D^{*0} \rangle \\
| (c \bar{d})_{J=1} \rangle &= | D^{*+} \rangle \\
| (c \bar{s})_{J=1} \rangle &= | D_s^{*} \rangle
\end{align*}
\]
Mesonic Ground States

**Bottom:**

\[
\begin{align*}
|({\bar{b}}u)_{J=0}\rangle &= |B^{-}\rangle \\
|({\bar{b}}d)_{J=0}\rangle &= |B^{0}\rangle \\
|({\bar{b}}s)_{J=0}\rangle &= |B_{s}^{-}\rangle \\
|({\bar{b}}u)_{J=1}\rangle &= |B^{*-}\rangle \\
|({\bar{b}}d)_{J=1}\rangle &= |B^{*0}\rangle \\
|({\bar{b}}s)_{J=1}\rangle &= |B_{s}^{*-}\rangle
\end{align*}
\]

**Charm:**

\[
\begin{align*}
|({\bar{c}}u)_{J=0}\rangle &= |D^{0}\rangle \\
|({\bar{c}}d)_{J=0}\rangle &= |D^{+}\rangle \\
|({\bar{c}}s)_{J=0}\rangle &= |D_{s}\rangle \\
|({\bar{c}}u)_{J=1}\rangle &= |D^{*0}\rangle \\
|({\bar{c}}d)_{J=1}\rangle &= |D^{*+}\rangle \\
|({\bar{c}}s)_{J=1}\rangle &= |D_{s}^{*}\rangle
\end{align*}
\]
Mesonic Ground States

**Bottom:**

\[
\begin{align*}
| (b\bar{u})_{J=0} \rangle &= |B^- \rangle \\
| (b\bar{d})_{J=0} \rangle &= |B^0 \rangle \\
| (b\bar{s})_{J=0} \rangle &= |B_s \rangle \\
| (b\bar{u})_{J=1} \rangle &= |B^{*-} \rangle \\
| (b\bar{d})_{J=1} \rangle &= |\bar{B}^*0 \rangle \\
| (b\bar{s})_{J=1} \rangle &= |\bar{B}^*_s \rangle
\end{align*}
\]

**Charm:**

\[
\begin{align*}
| (c\bar{u})_{J=0} \rangle &= |D^0 \rangle \\
| (c\bar{d})_{J=0} \rangle &= |D^+ \rangle \\
| (c\bar{s})_{J=0} \rangle &= |D_s \rangle \\
| (c\bar{u})_{J=1} \rangle &= |D^{*0} \rangle \\
| (c\bar{d})_{J=1} \rangle &= |D^{*+} \rangle \\
| (c\bar{s})_{J=1} \rangle &= |D^{*}_s \rangle
\end{align*}
\]
Mesonic Ground States

Bottom:

\(|(b\bar{u})_{J=0}\rangle = |B^-\rangle\)

\(|(b\bar{d})_{J=0}\rangle = |\bar{B}^0\rangle\)

\(|(b\bar{s})_{J=0}\rangle = |\bar{B}_s\rangle\)

\(|(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle\)

\(|(b\bar{d})_{J=1}\rangle = |\bar{B}^{*0}\rangle\)

\(|(b\bar{s})_{J=1}\rangle = |\bar{B}_s^*\rangle\)

Charm:

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Mesonic Ground States

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\[ |(b\bar{s})_{J=0}\rangle = |B_s\rangle \]
\[ |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle \]
\[ |(b\bar{d})_{J=1}\rangle = |B^*\rangle \]
\[ |(b\bar{s})_{J=1}\rangle = |B_s^*\rangle \]

Charm:

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\[ |(c\bar{s})_{J=1}\rangle = |D_s^*\rangle \]
Mesonic Ground States

Bottom:

\[ |(b\bar{u})_{J=0}\rangle = |B^-\rangle \quad |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle \]
\[ |(b\bar{d})_{J=0}\rangle = |B^0\rangle \quad |(b\bar{d})_{J=1}\rangle = |B^{*0}\rangle \]
\[ |(b\bar{s})_{J=0}\rangle = |B_s\rangle \quad |(b\bar{s})_{J=1}\rangle = |B_s^{*-}\rangle \]

Charm:

\[ |(c\bar{u})_{J=0}\rangle = |D^0\rangle \quad |(c\bar{u})_{J=1}\rangle = |D^{*0}\rangle \]
\[ |(c\bar{d})_{J=0}\rangle = |D^+\rangle \quad |(c\bar{d})_{J=1}\rangle = |D^{*+}\rangle \]
\[ |(c\bar{s})_{J=0}\rangle = |D_s\rangle \quad |(c\bar{s})_{J=1}\rangle = |D_s^{*-}\rangle \]
Mesonic Ground States

Bottom:

\[ |(b\bar{u})_{J=0}\rangle = |B^-\rangle \]
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\[ |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle \]
\[ |(b\bar{d})_{J=1}\rangle = |\bar{B}^{*0}\rangle \]
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Charm:

\[ |(c\bar{u})_{J=0}\rangle = |D^0\rangle \]
\[ |(c\bar{d})_{J=0}\rangle = |D^+\rangle \]
\[ |(c\bar{s})_{J=0}\rangle = |D_s\rangle \]
\[ |(c\bar{u})_{J=1}\rangle = |D^{*0}\rangle \]
\[ |(c\bar{d})_{J=1}\rangle = |D^{*+}\rangle \]
\[ |(c\bar{s})_{J=1}\rangle = |D_s^*\rangle \]
Baryonic Ground States

\[
\begin{align*}
|\,(ud)_0\,1/2\rangle &= |\Lambda_Q\rangle \\
|\,(uu)_1\,1/2\rangle, |\,(ud)_1\,1/2\rangle, |\,(dd)_1\,1/2\rangle &= |\Sigma_Q\rangle \\
|\,(uu)_1\,3/2\rangle, |\,(ud)_1\,3/2\rangle, |\,(dd)_1\,3/2\rangle &= |\Sigma^*_Q\rangle \\
|\,(us)_0\,1/2\rangle, |\,(ds)_0\,1/2\rangle &= |\Xi_Q\rangle \\
|\,(us)_1\,1/2\rangle, |\,(ds)_1\,1/2\rangle &= |\Xi'_Q\rangle \\
|\,(us)_1\,3/2\rangle, |\,(ds)_1\,3/2\rangle &= |\Xi^*_Q\rangle \\
|\,(ss)_1\,1/2\rangle &= |\Omega_Q\rangle \\
|\,(ss)_1\,3/2\rangle &= |\Omega^*_Q\rangle
\end{align*}
\]
Baryonic Ground States

\[
\begin{align*}
\left| \left[ (ud)_{0}Q \right]_{1/2} \right> &= |\Lambda_{Q} \rangle \\
\left| \left[ (uu)_{1}Q \right]_{1/2} \right> , \left| \left[ (ud)_{1}Q \right]_{1/2} \right> , \left| \left[ (dd)_{1}Q \right]_{1/2} \right> &= |\Sigma_{Q} \rangle \\
\left| \left[ (uu)_{1}Q \right]_{3/2} \right> , \left| \left[ (ud)_{1}Q \right]_{3/2} \right> , \left| \left[ (dd)_{1}Q \right]_{3/2} \right> &= |\Sigma^{*}_{Q} \rangle \\
\left| \left[ (us)_{0}Q \right]_{1/2} \right> , \left| \left[ (ds)_{0}Q \right]_{1/2} \right> &= |\Xi_{Q} \rangle \\
\left| \left[ (us)_{1}Q \right]_{1/2} \right> , \left| \left[ (ds)_{1}Q \right]_{1/2} \right> &= |\Xi^{*}_{Q} \rangle \\
\left| \left[ (us)_{1}Q \right]_{3/2} \right> , \left| \left[ (ds)_{1}Q \right]_{3/2} \right> &= |\Xi_{Q}^{*} \rangle \\
\left| \left[ (ss)_{1}Q \right]_{1/2} \right> &= |\Omega_{Q} \rangle \\
\left| \left[ (ss)_{1}Q \right]_{3/2} \right> &= |\Omega_{Q}^{*} \rangle 
\end{align*}
\]
Baryonic Ground States

\[
\begin{align*}
\left[(ud)_0 Q\right]_{1/2} & = |\Lambda_Q\rangle \\
\left[(uu)_1 Q\right]_{1/2} & , \left[(ud)_1 Q\right]_{1/2} & , \left[(dd)_1 Q\right]_{1/2} & = |\Sigma_Q\rangle \\
\left[(uu)_1 Q\right]_{3/2} & , \left[(ud)_1 Q\right]_{3/2} & , \left[(dd)_1 Q\right]_{3/2} & = |\Sigma^*_Q\rangle \\
\left[(us)_0 Q\right]_{1/2} & , \left[(ds)_0 Q\right]_{1/2} & = |\Xi_Q\rangle \\
\left[(us)_1 Q\right]_{1/2} & , \left[(ds)_1 Q\right]_{1/2} & = |\Xi'_Q\rangle \\
\left[(us)_1 Q\right]_{3/2} & , \left[(ds)_1 Q\right]_{3/2} & = |\Xi^*_Q\rangle \\
\left[(ss)_1 Q\right]_{1/2} & = |\Omega_Q\rangle & \left[(ss)_1 Q\right]_{3/2} & = |\Omega^*_Q\rangle
\end{align*}
\]
Baryonic Ground States

\[ |(ud)_0 Q\rangle_{1/2} = |\Lambda_Q\rangle \]
\[ |(uu)_1 Q\rangle_{1/2}, |(ud)_1 Q\rangle_{1/2}, |(dd)_1 Q\rangle_{1/2} = |\Sigma_Q\rangle \]
\[ |(uu)_{1/2} Q\rangle, |(ud)_{1/2} Q\rangle, |(dd)_{1/2} Q\rangle = |\Sigma^*_Q\rangle \]
\[ |(us)_0 Q\rangle_{1/2}, |(ds)_0 Q\rangle_{1/2} = |\Xi_Q\rangle \]
\[ |(us)_1 Q\rangle_{1/2}, |(ds)_1 Q\rangle_{1/2} = |\Xi'_Q\rangle \]
\[ |(us)_{1/2} Q\rangle, |(ds)_{1/2} Q\rangle = |\Xi^*_Q\rangle \]
\[ |(ss)_1 Q\rangle_{1/2} = |\Omega_Q\rangle \]
\[ |(ss)_{1/2} Q\rangle, |(ss)_{3/2} Q\rangle = |\Omega^*_Q\rangle \]
**Motivation**

Effective Field Theories

**Heavy mass limit**

**Heavy Quark Effective Theory**

Determination of $V_{cb}$

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**Baryonic Ground States**

\[ |(ud)_0 Q\rangle_{1/2} = |\Lambda_Q\rangle \]

\[ |(uu)_1 Q\rangle_{1/2}, |(ud)_1 Q\rangle_{1/2}, |(dd)_1 Q\rangle_{1/2} = |\Sigma_Q\rangle \]

\[ |(uu)_1 Q\rangle_{3/2}, |(ud)_1 Q\rangle_{3/2}, |(dd)_1 Q\rangle_{3/2} = |\Sigma^*_Q\rangle \]

\[ |(us)_0 Q\rangle_{1/2}, |(ds)_0 Q\rangle_{1/2} = |\Xi_Q\rangle \]

\[ |(us)_1 Q\rangle_{1/2}, |(ds)_1 Q\rangle_{1/2} = |\Xi'_Q\rangle \]

\[ |(us)_1 Q\rangle_{3/2}, |(ds)_1 Q\rangle_{3/2} = |\Xi^*_Q\rangle \]

\[ |(ss)_1 Q\rangle_{1/2} = |\Omega_Q\rangle \]

\[ |(ss)_1 Q\rangle_{3/2} = |\Omega^*_Q\rangle \]
Baryonic Ground States

\[
\begin{align*}
|\langle (ud)_0 Q \rangle_{1/2} \rangle &= |\Lambda_Q \rangle \\
|\langle (uu)_1 Q \rangle_{1/2} \rangle , |\langle (ud)_1 Q \rangle_{1/2} \rangle , |\langle (dd)_1 Q \rangle_{1/2} \rangle &= |\Sigma_Q \rangle \\
|\langle (uu)_1 Q \rangle_{3/2} \rangle , |\langle (ud)_1 Q \rangle_{3/2} \rangle , |\langle (dd)_1 Q \rangle_{3/2} \rangle &= |\Sigma^*_Q \rangle \\
|\langle (us)_0 Q \rangle_{1/2} \rangle , |\langle (ds)_0 Q \rangle_{1/2} \rangle &= |\Xi_Q \rangle \\
|\langle (us)_1 Q \rangle_{1/2} \rangle , |\langle (ds)_1 Q \rangle_{1/2} \rangle &= |\Xi'_Q \rangle \\
|\langle (us)_1 Q \rangle_{3/2} \rangle , |\langle (ds)_1 Q \rangle_{3/2} \rangle &= |\Xi^*_Q \rangle \\
|\langle (ss)_1 Q \rangle_{1/2} \rangle &= |\Omega_Q \rangle \\
|\langle (ss)_1 Q \rangle_{3/2} \rangle &= |\Omega^*_Q \rangle
\end{align*}
\]
Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

\[ \langle H^(*) (\nu) | Q_\nu \Gamma Q_{\nu'} | H^(*) (\nu') \rangle = C_\Gamma (\nu, \nu') \xi (\nu \cdot \nu') \]

with \( H^(*) (\nu) = D^(*) (\nu) \) or \( B^(*) (\nu) \)

- \( C_\Gamma (\nu, \nu') \): Computable Clebsh Gordan Coefficient
- \( \xi (\nu \cdot \nu') \): Reduced Matrix Element
- \( \xi (\nu \cdot \nu') \): universal non-perturbative Form Faktor: Isgur Wise Funktion
- Normalization of \( \xi \) at \( \nu = \nu' \):

\[ \xi (\nu \cdot \nu' = 1) = 1 \]
Representations for ground state meson

**Spin 1/2 of heavy quark \times Spin 1/2 of light anti-quark**

- \( \rightarrow 0^- \) state

\[
\sum_{s_h, s_l} C_{CG}(0; s_h, s_l) u^\text{heavy}_\alpha(v, s_h) \bar{v}^\text{light}_\beta(v, s_l) = \mathcal{N} \left[ (\psi + 1) \gamma^5 \right]_{\alpha\beta}
\]

- \( \rightarrow 1^- \) state, Polarization \( \epsilon \quad (v \cdot \epsilon = 0) \)

\[
\sum_{s_h, s_l} C_{CG}(0; s_h, s_l) u^\text{heavy}_\alpha(v, s_h) \bar{v}^\text{light}_\beta(v, s_l) = \mathcal{N} \left[ (\psi + 1) \epsilon' \right]_{\alpha\beta}
\]
Calculation of the Coefficients:

\[
\langle H(v) | Q_v \Gamma Q_{v'} | H(v') \rangle = \mathcal{N}^2 \text{Tr} \{ \gamma_5 (\not{v} + 1) \Gamma (\not{v'} + 1) \gamma_5 \} \xi (v \cdot v')
\]

\[
\langle H^* (v, \epsilon) | Q_v \Gamma Q_{v'} | H(v') \rangle = \mathcal{N}^2 \text{Tr} \{ \not{\epsilon} (\not{v} + 1) \Gamma (\not{v'} + 1) \gamma_5 \} \xi (v \cdot v')
\]

\[
\langle H^* (v, \epsilon) | Q_v \Gamma Q_{v'} | H(v', \epsilon') \rangle = \mathcal{N}^2 \text{Tr} \{ \not{\epsilon} (\not{v} + 1) \Gamma (\not{v'} + 1) \not{\epsilon}' \} \xi (v \cdot v')
\]
The heavy mass limit can be formulated as an effective field theory.

Expansion in inverse powers of $m_Q$

Define the static field $h_v$ for the velocity $v$

$$h_v(x) = e^{im_Qv \cdot x} \frac{1}{2} (1 + \gamma) b(x) \quad \rho_Q = m_Q v + k$$

HQET Lagrangian

$$\mathcal{L} = \bar{h}_v (iv \cdot D) h_v + \frac{1}{2m_Q} \bar{h}_v (i\not{D})^2 h_v + \cdots$$

Dim-4 Term: Feynman rules, loops, renormalization...
Example: $b \rightarrow c$ current

- At scales above $m_b$: Bottom and charm dynamical:
  - Full QCD
  - No anomalous dimension

- At scales between $m_b$ and $m_c$: Bottom static

- At scales below $m_c$: Bottom and charm static

\[
\gamma = \frac{\alpha_s}{\pi} f(vv')
\]
**Determination of $V_{cb}$ : $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$**

- Kinematic variable for a heavy quark: Four Velocity $v$
- Differential Rates

\[
\frac{d\Gamma}{d\omega}(B \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega)(F(\omega))^2
\]
\[
\frac{d\Gamma}{d\omega}(B \rightarrow D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (G(\omega))^2
\]

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $F$ and $G$: Form Factors
Heavy Quark Symmetries

- Normalization of the Form Factors is known at $\nu\nu' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

\[
\mathcal{F}(\omega) = \eta_{\text{QED}} \eta_A \left[ 1 + \delta_1/\mu^2 + \cdots \right] (\omega - 1) \rho^2 + \mathcal{O}((\omega - 1)^2)
\]

\[
\mathcal{G}(1) = \eta_{\text{QED}} \eta_V \left[ 1 + \mathcal{O} \left( \frac{m_B - m_D}{m_B + m_D} \right) \right]
\]

- Parameter of HQS breaking: $\mu = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$,
- $\delta_1/\mu^2 = -(8 \pm 4)\%$, $\eta_{\text{QED}} = 1.007$
Unquenched Calculations become available!
Heavy Mass Limit is not used
Lattice Calculations of the deviation from unity

$$\mathcal{F}(1) = 0.91^{+0.03}_{-0.04}$$

$$\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016$$

A. Kronfeld et al.
Motivation
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Determinations of $V_{cb}$

$B \rightarrow D^* \ell \bar{\nu}_\ell$

Thomas Mannel, CERN-PH-TH / University of Siegen
Part I: HQET
**Motivation**

Effective Field Theories

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**Heavy Quark Effective Theory**

Determination of $V_{cb}$

**$B \rightarrow D\ell\bar{\nu}_\ell$**

- ALEPH
  - 38.85 ± 11.80 ± 6.20

- CLEO
  - 44.77 ± 5.90 ± 3.45

- BELLE
  - 41.11 ± 4.40 ± 5.15

- Average
  - 42.30 ± 4.50

**HFAG LP 2007**

$\chi^2$/dof = 0.26/ 4 (CL = 99 %)
Motivation
Effective Field Theories
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Heavy Quark Effective Theory
Determination of $V_{cb}$

$$V_{cb, excl} = (38.6 \pm 1.3) \times 10^{-3}$$

- Potentially large $1/m_c^2$ terms
- Hints that $\mathcal{F}(1) \leq 0.89$ (see below)

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